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Corporate Debt Maturity and Business Cycle Fluctuations*

Francesco Ferrante, Andrea Prestipino, Immo Schott

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Abstract

Long-term debt is the main source of firm-financing in the U.S. We show that accounting for debt maturity is crucial for understanding business cycle dynamics. We develop a macroeconomic model with defaultable long-term debt and equity adjustment costs. With long-term debt, firms have an incentive to increase leverage in order to dilute the value of outstanding debt. When equity issuance is costly, this incentive helps firms raise more debt through a *debt dilution channel* and mitigates the decline in net worth through a *balance sheet channel*, dampening the decline in investment in response to a negative financial shock. Using firm-level data, we estimate equity issuance costs and incorporate our findings into an estimated medium-scale DSGE model. Accounting for debt maturity and the cost of equity financing implies that credit supply shocks are the primary drivers of business cycle fluctuations.

JEL classification: E32, E44, E51
Keywords: Long-term debt; Financial frictions; Debt overhang; Macroeconomic activity.

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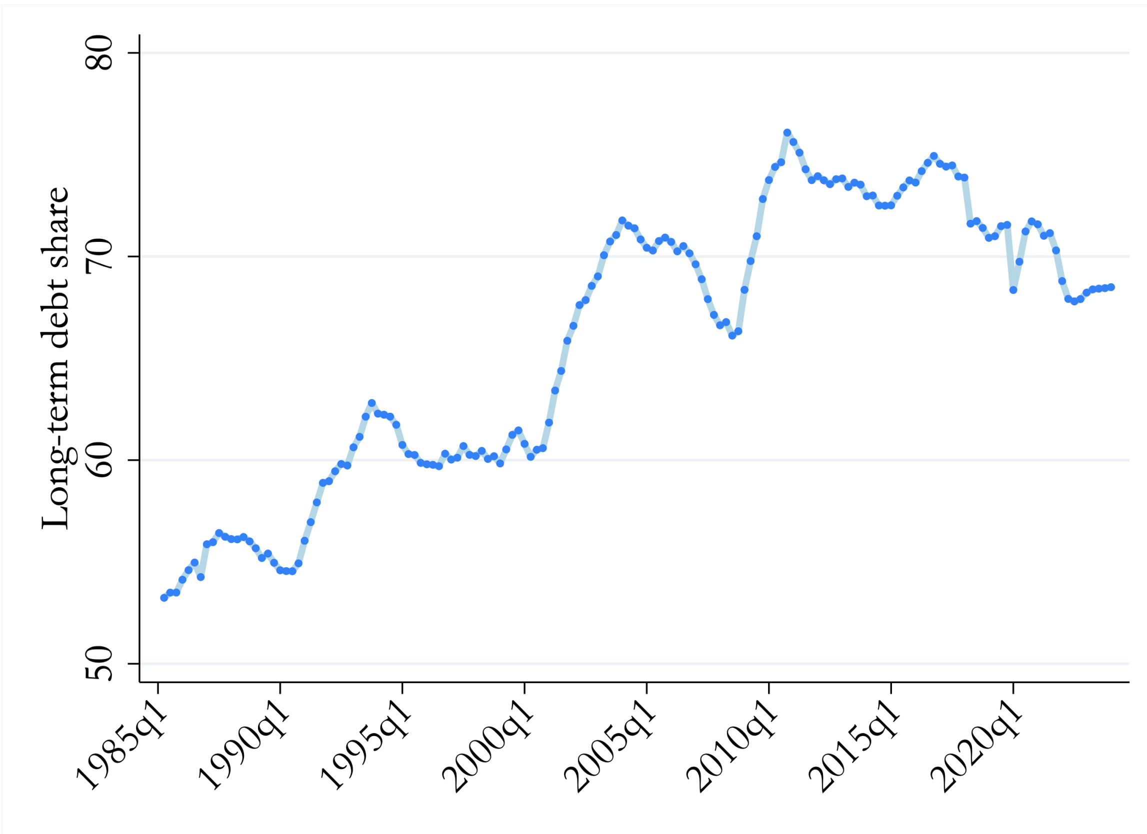
1 Introduction

Figure 1 shows that the vast majority of U.S. non-financial corporate debt is long-term. The strong reliance on long-term financing raises important questions about the role of firms' debt maturity in determining the sensitivity of the economy to shocks.

Figure 1: Long-term debt share



Note.- Figure shows the share of debt maturing in more than one year as a percentage of total debt for non-financial corporate businesses. Source: US Financial Accounts.

Traditional macroeconomic models of firm financing, such as Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Moore (1997), focus on how financial frictions can amplify the effects of monetary policy and other shocks. As investment and the price of capital decline, the balance sheets of borrowers deteriorate, financial constraints tighten, and investment falls further through a *financial accelerator* channel. These models focus on short-term debt and do not allow firms to issue new equity.

On the other hand, the corporate finance literature has studied how long-term debt can generate under-investment through a *debt overhang* channel (Myers, 1977; Hennessy, 2004): Equity investors inject less capital when the returns on investment primarily benefit existing creditors. Gomes, Jermann, and Schmid (2016) incorporate nominal, defaultable long-term debt into a macroeconomic model and show how this channel can amplify aggregate fluctuations. The presence of outstanding long-term debt creates a commitment problem for the firm which generates an incentive to increase leverage in order to dilute the value of outstanding debt. This mechanism leads to persistently elevated default rates, lower equity financing, and a stronger contraction in investment in response to standard macroeconomic shocks. However, because Gomes et al. (2016) assume that firms can issue equity without any cost, they do not allow for a financial accelerator channel.

Our paper bridges the gap between these two strands of literature by developing a model in which firms finance investment with defaultable long-term debt *and* costly equity issuance. We use this model to make three contributions. First, we show that equity adjustment costs are key in determining the role of long-term debt in transmitting shocks: When equity adjustments are costless, long-term debt amplifies the response of the economy to shocks, whereas the effect of shocks is dampened when it is costly to adjust equity. Second, using balance sheet data for U.S. firms, we estimate equity adjustment costs to be substantial. Third, we embed our mechanism into an estimated medium-scale DSGE model and show that including the observed maturity structure of corporate debt together with realistic equity adjustment costs leads us to reassess the drivers of business cycle fluctuations. In particular, we find that credit supply shocks are the main driver of business cycle fluctuations.

We now turn to a more detailed discussion of these contributions. In our framework, long-term debt and equity issuance costs interact to shape the response of the economy to shocks. We illustrate this point by considering two benchmarks that have been studied in the literature: one in which equity issuance is costless and one in which firms cannot issue equity at all. In line with Gomes et al. (2016), we find that when equity issuance is costless, a shock that increases the real value of firms debt leads to a decline in investment and output only when debt is long-term. The higher real burden of debt increases firms' incentive to dilute preexisting creditors by increasing leverage, resulting in higher default rates and lower investment through a *debt overhang* channel. When considering the same shock in an economy where firms cannot issue equity, we find that an increase in the real value of debt leads to a drop in net worth and in investment through a financial accelerator channel, both with short and long-term debt. However, long-term debt now plays a dampening role: the increase in leverage and the lower price of outstanding debt obligations, caused by the debt dilution motive, increase net worth and result in higher investment. A novel contribution of our paper is to show that the same debt dilution incentive that is responsible for the drop in investment without equity adjustment costs, dampens the decline in investment when equity issuance is costly.

In a next step we use balance sheet data from Compustat together with the model's optimality conditions to estimate the costs of adjusting equity. In line with the evidence from the corporate finance literature (e.g., Hennessy and Whited (2007)) we find that these costs are large, suggesting that costly equity injections are important for shaping firms' investment decisions.

Finally, in a quantitative exercise, we embed our estimates into an estimated medium scale DSGE model with financial frictions similar to Christiano, Motto, and Rostagno (2014) and find that shocks to the supply of credit emerge as the main driver of business cycle fluctuations. The credit supply shock is the only shock that induces data-consistent co-movements between macroeconomic and financial variables. In contrast, other prominent shocks from the literature, i.e., both shocks to the marginal efficiency of investment (Justiniano, Primiceri, and Tambalotti, 2010) and risk shocks (Christiano et al., 2014) cause equity to increase in response to a recessionary shock. We show that the presence of long-term debt and costly equity issuance are two key ingredients for the inference regarding which shocks drive business cycle fluctuations. First, accounting for debt maturity is crucial for the risk shock to be driven out by the credit supply shock. With long-term debt, the risk shock increases firms' debt dilution incentives and causes the value of outstanding debt to decline. As a result, the price of capital declines by less than the price of debt, leading to a counterfactual increase in equity. On the other hand, with short-term debt an increase in idiosyncratic investment risk causes equity to decline because there is no valuation effect on previously issued debt.[1] Second, accounting for the substantial costs of adjusting equity is necessary for the credit supply shock to generate a recession. Without equity adjustment costs, firms can respond to a credit supply shock by rais-

[1]If we assume one-period debt - ignoring the actual maturity structure of corporate debt in the data - we find that risk shocks are the main driver of fluctuations in investment, in line with Christiano et al. (2014).

ing additional capital while reducing debt. This decline in leverage reduces the debt overhang effect and stimulates investment.

An additional contribution of our paper is methodological. The solution of models with long-term defaultable debt is complicated by the need to compute the derivative of the leverage policy function, which cannot be obtained with standard perturbation methods. We overcome this challenge by developing a novel algorithm which is based on a global solution of the problem at the steady state and a linear approximation of the derivative of the policy function with respect to the aggregate macro variables. This strategy makes Bayesian estimation of our DSGE model feasible.

Related literature By studying how the interaction of long-term corporate debt and financial frictions shapes the response to macroeconomic shocks in a medium-scale DSGE model, our work contributes to three strands of literature.

First, a large literature has studied the implications of corporate debt for macroeconomic fluctuations. These models typically focus on one-period debt and thus abstract from any frictions related to debt with long maturity (e.g. Bernanke et al. (1999), Kiyotaki and Moore (1997), Christiano et al. (2014), Arellano, Bai, and Kehoe (2019)). Furthermore, firms typically cannot raise equity in these models.[2] Papers in this literature include a financial accelerator mechanism which links firm investment to changes in net worth, but abstract from the interaction between the value of long-term debt and firms' balance sheets.[3]

Second, we contribute to the literature on the effects of defaultable long-term corporate debt. Miao and Wang (2010) incorporate long-term defaultable corporate bonds and credit risk in a dynamic stochastic general equilibrium business cycle model and show that credit risk amplifies aggregate fluctuations. Gomes et al. (2016) show that unanticipated changes in inflation can have persistent effects on aggregate investment by affecting the real burden of long-term nominal debt. Relatedly, Jungherr and Schott (2022) show that risky long-term

[2]Notable exceptions include Jermann and Quadrini (2012), Gertler, Kiyotaki, and Queralto (2012), and Gertler, Kiyotaki, and Prestipino (2020a), which feature short-term debt and (costly) equity adjustments. Ferrante (2019) features long-term debt but assumes infinitely costly equity issuance.

[3]A related literature focuses on firm heterogeneity. Jungherr, Meier, Reinelt, and Schott (2024) show that investment of firms with a larger amount of maturing debt is more responsive to monetary policy and rationalize this finding with a heterogeneous firm model with defaultable long-term debt and costly equity adjustments. In addition, Brunnermeier, Correia, Luck, Verner, and Zimmermann (2023) show that firms with a larger share of long-term debt were more resilient during the German hyperinflation period of 1919-1923.

debt at the firm-level can lead to an amplified investment response to technology shocks and thereby rationalize the slow adjustment of aggregate leverage during recessions. These papers highlight the distortionary effect of the debt overhang channel on firms' investment decisions but abstract from equity issuance costs. Our paper instead focuses on the interplay between long-term debt and firms' net worth when equity is costly to adjust.[4]

A third strand of literature investigates the drivers of U.S. business cycle fluctuations in macroeconomic models. Smets and Wouters (2007) were among the first to perform this type of analysis and found that, in a medium scale DSGE model, demand shocks (like a risk premium shock or a government spending shock) accounted for the bulk of short-run output fluctuations, whereas supply shocks played an important role for output variations in the medium- to long-run. Justiniano et al. (2010) found shocks to the marginal efficiency of investment to be the driving force of the business cycle. Christiano et al. (2014) revisit this result in a model with financial frictions. They show that once financial variables are included in the estimation, a risk shock that affects the volatility of idiosyncratic uncertainty explains most of the variation in output. We contribute to this literature by showing that the implications of the risk shock crucially depend on corporate debt maturity, and by studying the important role played by a credit supply shock in accounting for business cycle fluctuations. Our results are consistent with work by Justiniano, Primiceri, and Tambalotti (2019) and Gertler, Kiyotaki, and Prestipino (2020b) who emphasize the role of credit supply shocks in explaining the Great Recession.

The structure of the paper is as follows. Section 2 describes the model. Section 3 explains the interaction between equity adjustment costs and debt maturity in determining the response of the economy to financial shocks. Section 4 contains our quantitative exercises. Section 5 concludes.

2 Model

There is a continuum of households, each consisting of a continuum of members. At each point in time a proportion e of household members are entrepreneurs, a proportion b are bankers, and the remaining proportion $1 - e - b$ are workers. We describe the optimization problems of

[4]The interaction between debt maturity and default has also been studied in the sovereign debt literature. See, for example, Hatchondo, Martinez, and Sosa-Padilla (2016), Aguiar, Amador, Hopenhayn, and Werning (2019) and Bocola and Dovis (2019).

these three types of agents in turn.

2.1 Workers

Workers choose household consumption, C_t, banks deposits, D_t, and government bonds, B_t, to maximize

$$\sum_{t=0}^{\infty} \beta^t \left[\xi_t^c \log\left(C_t - hC_{t-1}\right) - \frac{\bar{\varphi}}{1+\varphi} \int h_{it}^{1+\varphi} di \right],$$

where ξ_t^c represents a preference shock. Workers' budget constraint is given by

$$C_t + D_t + B_t = \int w_{it} h_{it} di + D_{t-1} \frac{R_{t-1}^d}{\pi_t} + B_{t-1} \frac{R_{t-1}}{\pi_t} + T_t,$$

where R_{t-1}^d and R_{t-1} are the nominal rates of return on deposits and government bonds between time $t-1$ and t, π_t is the rate of inflation, and T_t collects all transfers to the household from firms, entrepreneurs, bankers, and the government. Workers take as given the choice of individual labor supply h_t^i and wages w_t^i, which are determined by labor agencies and labor unions as described in section 2.8.

Households' optimal holdings of deposits and government bonds satisfy

$$1 = \mathbb{E}_t \Lambda_{t+1} \frac{R_t^d}{\pi_{t+1}} = \mathbb{E}_t \Lambda_{t+1} \frac{R_t}{\pi_{t+1}}, \tag{1}$$

where $\Lambda_{t+1} = \beta \frac{U_c(t+1)}{U_c(t)}$ is the stochastic discount factor with the marginal utility of consumption given by

$$U_c\left(t\right) = \frac{\xi_t^c}{C_t - hC_{t-1}} - \beta h \mathbb{E}_t \frac{\xi_{t+1}^c}{C_{t+1} - hC_t}.$$

2.2 Entrepreneurs

Entrepreneurs hold the productive capital in the economy. We assume that entrepreneurs rent their capital to firms at a market rental rate r_t^k. An entrepreneur's return on capital investment is given by

$$\xi_t R_t^k = \xi_t \frac{r_t^k + (1-\delta) Q_t^k}{Q_{t-1}^k}, \tag{2}$$

where Q_t^k is the price of capital, δ is the depreciation rate, and ξ_t is an idiosyncratic shock. We assume that $\log(\xi_t)$ is distributed as a Normal with mean zero and standard deviation σ_{t-1}, allowing for "risk shocks", i.e. exogenous fluctuations in σ_t.[5]

Entrepreneurs finance capital purchases with equity, x_t, and by issuing long-term nominal debt l_t. Their flow budget constraint at time t is

$$Q_t^k k_t = x_t + Q_t^l l_t,$$

where Q_t^l is the price of debt.[6] Each unit of debt issued at $t-1$ pays a coupon c_l at time t, while the remaining portion $1-\lambda_l$ remains outstanding. The parameter λ_l thus controls the maturity of corporate debt. When $\lambda_l = 1$, debt is short term whereas when $\lambda_l < 1$ debt maturity is greater than a quarter. The firm can default on its debt obligations. In that case, the entrepreneur exits the market and its creditors take over the productive capital, subject to default costs, as discussed in section 2.4 below.

The net worth of an entrepreneur who owns k_{t-1} units of capital and has outstanding debt of l_{t-1} at time t is given by

$$n_t = \xi_t R_t^k Q_{t-1}^k k_{t-1} - \frac{c_l + (1-\lambda_l)\, Q_t^l}{\pi_t} l_{t-1}. \tag{3}$$

Each entrepreneur chooses a fraction $1-\omega_t$ of net worth to distribute to households as dividends. Equity is given by $x_t = \Psi(\omega_t) n_t < \omega_t n_t$, where $\Psi(\omega_t)$ includes equity injection costs:

$$\Psi(\omega_t) = (1-\overline{\psi})\omega_t - \frac{\psi}{2}(\omega_t - \overline{\omega})^2.$$

Here, $\overline{\psi} \geq 0$, $\psi \geq 0$, and $\overline{\omega}$ is a target equity injection rate. This formulation of equity injection costs is a simple way to capture agency problems that limit entrepreneurs' ability to substitute debt- with equity finance.[7] The parameter $\bar{\psi}$ represents a linear cost which guarantees that around the steady state of the model entrepreneurs have an incentive to use debt to finance capital purchases.[8] Without equity issuance costs, i.e., $\bar{\psi} = 0$ and $\psi = 0$, entrepreneurs are able to costlessly substitute debt- with equity finance in response to changing financial conditions,

[5] This shock was introduced by Williamson (1987) and its properties were studied more recently by Christiano et al. (2014).

[6] For ease of notation, we are omitting the dependence of the price of debt on the entrepreneur's financial decisions. We will be explicit about it when describing the entrepreneur's optimality conditions.

[7] A similar approach to model costly dividend payouts is used in Jermann and Quadrini (2012). The costs are proportional to net worth to obtain aggregation of entrepreneurs' policy functions, as in Elenev, Landvoigt, and Van Nieuwerburgh (2021).

[8] This cost plays a role similar to a tax advantage of debt, used, for example, in Gomes et al. (2016).

as is assumed, for instance, in Gomes et al. (2016). At the other extreme, when no new equity can be issued, i.e., $\psi = \infty$, entrepreneurs always retain a fixed share of net worth $\overline{\omega}$ as equity in the firm, and their only active margin of financial adjustment is through debt. This is the assumption in most papers with financial frictions following Bernanke et al. (1999). We allow for ψ to take on any positive value and estimate its value in Section 4.2.

Let $V_t(k_{t-1}, l_{t-1}, \xi_t)$ be the value function of a non-defaulting firm that enters period t with capital k_{t-1}, debt l_{t-1}, and an idiosyncratic shock ξ_t. The firm's value is given by the present discounted value of dividend payouts,

$$V_t\left(k_{t-1}, l_{t-1}, \xi_t\right) = \max_{k_t, l_t, \omega_t, n_t} \left(1-\omega_t\right) n_t + \mathbb{E}_t \Lambda_{t,t+1} \max\left\{0, V_{t+1}\left(k_t, l_t, \xi_{t+1}\right)\right\}, \tag{4}$$

subject to the definition of net worth (3) and the flow budget constraint

$$Q_t^k k_t = \Psi(\omega_t) n_t + Q_t^l l_t. \tag{5}$$

The value function is linear in k_{t-1}. To see this, let $\eta_t = \frac{l_t}{k_t}$ denote the entrepreneur's leverage ratio and define the per unit of capital value as $v_t(\eta_{t-1}, \xi_t) = \frac{V_t(k_{t-1}, l_{t-1}, \xi_t)}{k_{t-1}}$. Dividing equation (4) by k_{t-1} we get:

$$v_t\left(\eta_{t-1}, \xi_t\right) = \max_{\eta_t, \omega_t} \mu_t(\eta_t, \eta_{t-1}, \xi_t) \cdot \left[\left(1-\omega_t\right) + \frac{\Psi\left(\omega_t\right)}{Q_t^k - Q_t^l \eta_t} \mathbb{E}_t \Lambda_{t,t+1} \max\left\{0, v_{t+1}\left(\eta_t, \xi_{t+1}\right)\right\}\right], \tag{6}$$

where $\mu_t(\eta_t, \eta_{t-1}, \xi_t)$ is the entrepreneur's net worth per unit of capital, given by

$$\mu_t\left(\eta_t, \eta_{t-1}, \xi_t\right) \equiv \frac{n_t}{k_{t-1}} = \xi_t R_t^k Q_{t-1}^k - \frac{c_l + \left(1-\lambda_l\right) Q_t^l}{\pi_t} \eta_{t-1}, \tag{7}$$

and we are using equations (5) and (7) to get $\frac{k_t}{k_{t-1}} = \frac{\Psi(\omega_t)\mu_t}{Q_t^k - Q_t^l \eta_t}$. Note that with long term debt, i.e. $\lambda_l < 1$, an entrepreneur's net worth per unit of capital, μ_t, depends on their choice of leverage, η_t, through the effect of leverage on the price of debt, Q_t^l.[9]

From (6) and (7), the optimal value $v_t\left(\eta_{t-1}, \xi_t\right)$ is increasing in ξ_t. As a result, there exists a threshold value of the idiosyncratic shock to the return on capital, denoted as $\overline{\xi}_{t+1}$, such that a firm defaults when $\xi_{t+1} < \overline{\xi}_{t+1}$. The threshold value satisfies $v_{t+1}\left(\eta_t, \overline{\xi}_{t+1}\right) = 0$, which using

[9] By contrast, with short term debt, i.e. $\lambda_l = 1$, net worth per unit of capital does not depend on an entrepreneur's choices at time t. In this case, μ_t just falls out of the optimization problem.

(7), pins down $\bar{\xi}_{t+1}$ as the value of the idiosyncratic shock that makes net worth equal to zero:

$$\bar{\xi}_{t+1}=\frac{1}{R^k_{t+1}Q^k_t}\cdot\frac{c_l+(1-\lambda_l)\,Q^l_{t+1}}{\pi_{t+1}}\eta_t. \tag{8}$$

Equity injections The optimality condition for equity injections is given by the derivative of the value function in (6) with respect to ω_t:

$$\frac{1}{\Psi'(\omega_t)}=\frac{\mathbb{E}_t\Lambda_{t,t+1}\int_{\bar{\xi}_{t+1}}^{\infty}v_{t+1}(\eta_t,\xi_{t+1})\,dF_t(\xi_{t+1})}{Q^k_t-Q^l_t\eta_t}=\gamma_t \tag{9}$$

It states that the marginal cost of equity, $\frac{1}{\Psi'(\omega_t)}$, is equal to the expected discounted return on equity injections, which we denote by γ_t. The term F_t is the distribution of idiosyncratic shocks at time $t+1$. To characterize the return γ_t further, it is useful to introduce the entrepreneur's Tobin's Q, i.e., the ratio between the value of a unit of net worth inside the firm and its book value, $\varphi_t\equiv\frac{v_t}{\mu_t}$. Using (6) and (8) we can write φ_t as

$$\varphi_t=(1-\omega_t)+\Psi(\omega_t)\frac{Q^k_t}{Q^k_t-Q^l_t\eta^l_t}\mathbb{E}_t\Lambda_{t,t+1}\varphi_{t+1}\int_{\bar{\xi}_{t+1}}^{\infty}\left(\xi_{t+1}-\bar{\xi}_{t+1}\right)R^k_{t+1}dF_t(\xi_{t+1}). \tag{10}$$

Using $v_{t+1}=\varphi_{t+1}\mu_{t+1}$ together with the definition of μ_{t+1} from (7) to substitute for v_{t+1} in (9), we can express the expected discounted returns on equity investment as

$$\gamma_t=\frac{Q^k_t}{Q^k_t-Q^l_t\eta_t}\mathbb{E}_t\Lambda_{t,t+1}\varphi_{t+1}\int_{\bar{\xi}_{t+1}}^{\infty}\left(\xi_{t+1}-\bar{\xi}_{t+1}\right)R^k_{t+1}dF_t(\xi_{t+1}), \tag{11}$$

which says that one unit of equity gets leveraged into $\frac{Q^k_t}{Q^k_t-Q^l_t\eta_t}$ units of capital, providing a return $\xi_{t+1}R^k_{t+1}$ net of the debt repayment $\bar{\xi}_{t+1}R^k_{t+1}$, whenever the idiosyncratic shock is larger than the threshold. The optimality condition for equity injection in (9) then becomes

$$\frac{1}{\Psi'(\omega_t)}=\frac{Q^k_t}{Q^k_t-Q^l_t\eta_t}\mathbb{E}_t\Lambda_{t,t+1}\varphi_{t+1}\int_{\bar{\xi}_{t+1}}^{\infty}\left(\xi_{t+1}-\bar{\xi}_{t+1}\right)R^k_{t+1}dF_t(\xi_{t+1}). \tag{12}$$

Leverage The optimality condition for leverage is given by

$$\frac{\partial\mu_t}{\partial\eta_t}\varphi_t+\frac{\Psi(\omega_t)\,\mu_t}{Q^k_t-Q^l_t\eta_t}\left[\left(Q^l_t+\frac{dQ^l_t}{d\eta_t}\eta_t\right)\gamma_t-E\Lambda_{t,t+1}\int_{\bar{\xi}_{t+1}}^{\infty}\frac{\partial v_{t+1}}{\partial\eta_t}dF_t(\xi_{t+1})\right]=0, \tag{13}$$

The first term in (13) is given by

$$\frac{\partial\mu_t}{\partial\eta_t}=-\frac{(1-\lambda_l)\,\eta_{t-1}}{\pi_t}\frac{dQ^l_t}{d\eta_t}. \tag{14}$$

This term captures a debt dilution incentive that works through the negative effect of an increase in leverage on the price of pre-existing debt, $\frac{dQ_t^l}{d\eta_t} < 0$. This incentive depends on the maturity of debt, λ_l, and disappears with short-term debt, i.e., $\lambda_l = 1$. In addition, the debt dilution incentive also increases with the marginal value of net worth, φ_t.

The second term, $\frac{\Psi(\omega_t)\mu_t}{Q_t^k - Q_t^l \eta_t^l}\left(Q_t^l + \frac{dQ_t^l}{d\eta_t}\eta_t^l\right)\gamma_t$, is the marginal benefit of increasing leverage associated with higher returns on capital, and the last term, $-\frac{\Psi(\omega_t)\mu_t}{Q_t^k - Q_t^l \eta_t^l} E\Lambda_{t,t+1}\int_{\bar{\xi}_{t+1}}^{\infty} \frac{\partial v_{t+1}}{\partial \eta_t} dF_t\left(\xi_{t+1}\right)$, captures the marginal cost of leverage. Using the envelope condition

$$\frac{dv_t}{d\eta_{t-1}} = -\frac{c_l + \left(1-\lambda_l\right) Q_t^l}{\pi_t}\varphi_t, \tag{15}$$

we can express the marginal cost of leverage as the cost of repaying the debt in case of non default:

$$\mathbb{E}_t\Lambda_{t,t+1}\int_{\bar{\xi}_{t+1}}^{\infty} \frac{\partial v_{t+1}}{\partial \eta_t} dF_t\left(\xi_{t+1}\right) = \mathbb{E}_t\Lambda_{t,t+1} Q_t^l R_{t+1}^l \varphi_{t+1}\left(1 - F_t\left(\bar{\xi}_{t+1}\right)\right),$$

where R_{t+1}^l is the real return on debt of non-defaulting firms:

$$R_{t+1}^l = \frac{1}{\pi_{t+1}} \cdot \frac{c_l + \left(1-\lambda_l\right) Q_{t+1}^l}{Q_t^l}. \tag{16}$$

Using (14) and (15) in (13) we can rewrite the optimality condition for entrepreneur's leverage as:

$$\mathbb{E}_t\Lambda_{t,t+1} R_{t+1}^l\left(1 - F_t\left(\bar{\xi}_{t+1}^l\right)\right)\varphi_{t+1} = \left(1+\epsilon_{\eta_t}\right)\gamma_t - \epsilon_{\eta_t}\frac{\left(1-\lambda_l\right)\eta_{t-1}}{\eta_t}\frac{\left(Q_t^k - Q_t^l \eta_t^l\right)}{\Psi\left(\omega_t\right)\mu_t}\frac{\varphi_t}{\pi_t}, \tag{17}$$

where $\epsilon_{\eta_t} = \frac{dQ_t^l}{d\eta_t}\frac{\eta_t}{Q_t^l} < 0$ is the elasticity of the price of debt with respect to leverage. Equations (8), (12) and (17), together with the definition of μ_t and φ_t in (7) and (10), are the optimality conditions for the entrepreneurs' problem. We now turn to describing aggregation of the entrepreneurs' policy functions.

2.3 Aggregation of entrepreneurs' choices

Defaulting entrepreneurs exit and are replaced by an equal measure of new entrepreneurs. These new entrants receive a transfer $T_t^e = \tau^e X_{t-1}$, in proportion to aggregate net worth, which is used to purchase capital:

$$Q_t^k k_t^e = T_t^e + Q_t^l l_t^e \tag{18}$$

To preserve aggregation we make two assumptions. First we assume that new entrants' leverage is the same as the leverage of existing entrepreneurs, implying $l_t^e = \eta_t k_t^e$. Second, to ensure that net worth per unit of capital, μ_t in (17), is constant among surviving firms, we assume that the ex-post return on capital is equal across non-defaulting entrepreneurs, i.e., $\tilde{\xi}_t = \frac{1}{1-F_{t-1}\left(\bar{\xi}_t\right)} \int_{\bar{\xi}_t} \xi_t dF_{t-1}\left(\xi_t\right)$, implying constant leverage choices across all entrepreneurs active at time t.[10]

Aggregate equity of entrepreneurs at time t is then given by

$$X_t = \Psi\left(\omega_t\right)\left(\tilde{\xi}_t - \bar{\xi}_t\right) R_t^k Q_{t-1}^k K_{t-1}\left(1 - F\left(\bar{\xi}_t^l\right)\right) + T_t^e + \xi_t^N, \tag{19}$$

which sums the equity of entrepreneurs that do not default at time t and the transfer to new entrepreneurs. The term ξ_t^N captures a net worth shock which exogenously affects aggregate entrepreneurial net worth.

Aggregate capital demand is given by

$$K_t = \frac{X_t}{Q_t^k - \eta_t Q_t^l}. \tag{20}$$

2.4 Financial intermediaries

Financial intermediaries raise deposits from households and invest these funds in corporate bonds according to

$$Q_t^l L_t = \kappa_t D_t, \tag{21}$$

where L_t and D_t represent aggregate bonds and deposits respectively, and κ_t is a credit supply shock that affects financial intermediaries' technology for transforming deposits into firm debt.

The expected return on corporate bonds, R_{t+1}^b, which takes into account the possibility of default, is given by

$$R_{t+1}^b = \left(1 - F_t\left(\bar{\xi}_{t+1}\right)\right) R_{t+1}^l + \gamma_l \frac{Q_t^k R_{t+1}^k}{\eta_t Q_t^l} \int_0^{\bar{\xi}_{t+1}} \xi_{t+1} dF_t\left(\xi_{t+1}\right), \tag{22}$$

where the second term represents the recovery value on defaulted debt, a fraction γ_l of the capital of defaulting firms. Because each creditor lends to a continuum of entrepreneurs, R_{t+1}^b

[10] This is achieved with a state-contingent transfer to each non-defaulting entrepreneur, i.e. an entrepreneur with $\xi_{i,t} > \bar{\xi}_t$, given by $(\tilde{\xi}_t - \xi_{i,t}) R_t^k Q_t^k k_{t-1}$. See Ferrante (2019) for details.

also represents the realized return on each creditors' debt portfolio.

Financial intermediaries are owned by households, and their objective is to maximize profits

$$\max_{D_t} D_t \cdot \mathbb{E}_t \Lambda_{t,t+1} \left(\kappa_t R^b_{t+1} - \frac{R^d_t}{\pi_{t+1}} \right),$$

Optimality implies

$$\mathbb{E}_t \Lambda_{t,t+1} \kappa_t R^b_{t+1} = \mathbb{E}_t \Lambda_{t,t+1} \frac{R^d_t}{\pi_{t+1}} = 1 \tag{23}$$

where the second equality comes from households' first order conditions. Multiplying (23) by Q^l_t we obtain an expression for entrepreneurs' price of debt:

$$Q^l_t = \mathbb{E}_t \Lambda_{t+1} \kappa_t \left\{ \left(1 - F_t\left(\bar{\xi}_{t+1}\right)\right) \frac{c_l + (1-\lambda_l) Q^l_{t+1}}{\pi_{t+1}} + \gamma_l \frac{Q^k_t R^k_{t+1}}{\eta_t} \int_0^{\bar{\xi}_{t+1}} \xi_{t+1} dF_t\left(\xi_{t+1}\right) \right\} \tag{24}$$

Equations (23) and (24) show that the credit supply shock κ_t introduces an exogenous wedge between the expected return on bonds and the risk-free rate. A lower κ_t increases the lenders' required rate of return on corporate bonds, reducing the supply of credit and putting downward pressure on Q^l_t.

We can decompose the bond spread, i.e. the difference between the rate of return on bonds of non-defaulting firms and the deposit rate, into a default premium and a liquidity premium:

$$\underbrace{\mathbb{E}_t(R^l_{t+1} - \frac{R^d_t}{\pi_{t+1}})}_{\text{Bond spread}} = \underbrace{\mathbb{E}_t(R^l_{t+1} - R^b_{t+1})}_{\text{Default premium}} + \underbrace{\mathbb{E}_t(R^b_{t+1} - \frac{R^d_t}{\pi_{t+1}})}_{\text{Liquidity premium}}. \tag{25}$$

The first component captures the expected default risk, which moves with aggregate shocks and is particularly sensitive to the risk shock σ_t, whereas the second component moves inversely with κ_t.[11]

Finally, using equation (24) we can compute the derivative of the debt price with respect to entrepreneurial leverage, which enters the first-order condition for leverage in (17) as

$$\frac{\partial Q^l_t}{\partial \eta_t} = -\mathbb{E}_t \Lambda_{t+1} \kappa_t \left\{ \frac{\partial \bar{\xi}_{t+1}}{\partial \eta_t} f_t\left(\bar{\xi}_{t+1}\right) \left[Q^l_t R^l_{t+1} - \gamma_l \frac{Q^k_t R^k_{t+1}}{\eta_t} \bar{\xi}_{t+1} \right] + \gamma_l \frac{Q^k_t R^k_{t+1}}{(\eta_t)^2} \int_0^{\bar{\xi}_{t+1}} \xi_{t+1} dF_t\left(\xi_{t+1}\right) \right\}$$
$$+ \mathbb{E}_t \Lambda_{t+1} \kappa_t [1 - F_t\left(\bar{\xi}_{t+1}\right)] (1-\lambda_l) \frac{\partial Q^l_{t+1}}{\partial \eta_{t+1}} \eta_{\eta,t+1}\left(\eta_t\right) \tag{26}$$

[11] A possible way to endogenize the liquidity premium would be to introduce an agency problem in the financial sector. See, for example Gertler and Karadi (2011) or Ferrante (2019).

The term in the first row of (26) includes the impact of entrepreneurs' leverage on next period default threshold and on the expected recovery rate in case of default. Because higher η_t results in a higher expected probability of default and in a lower recovery rate, this term is negative. The term on the second row of (26) captures the effect of current leverage on the choice of future leverage, as measured by the derivative of the entrepreneurs' leverage policy function $\eta_{\eta,t+1}(\eta_t) = \frac{\partial \eta_{t+1}}{\partial \eta_t}$.[12] As shown in (17), and as noted by Gomes et al. (2016), long-term debt introduces an incentive for entrepreneurs to dilute the value of preexisting debt by increasing leverage, and rational lenders take this effect into account. The computation of this derivative complicates the numerical solution of the model, because it cannot be computed with standard perturbation methods. As discussed below, to overcome this problem we develop an algorithm which uses global solution techniques to capture the local dynamics of this derivatives around the steady state of the model.

2.5 Final good producers

The final good Y_t is a CES composite of different intermediate varieties, given by

$$Y_t = \left[\int Y_t(i)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} \right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}}, \tag{27}$$

where ε_t is time-varying due to a markup shock. The demand for each variety will be given by

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_t} Y_t, \tag{28}$$

where the aggregate price level is given by

$$P_t = \left[\int (P_t(i))^{1-\varepsilon_t} \, di \right]^{\frac{1}{1-\varepsilon_t}}. \tag{29}$$

[12] We are abusing notation here since the policy function is the *individual* entrepreneur's policy function for leverage as a function of individual entrepreneur's leverage. See Appendix for details.

2.6 Intermediate goods producers

Intermediate goods producers hire labor at real wage w_t and rent capital at rate r_t^k to produce using a Cobb-Douglas technology

$$Y_t = A_t (u_t K_{t-1})^{\chi} H_t^{1-\chi}, \tag{30}$$

where A_t is aggregate TFP, and H_t and K_{t-1} are aggregate labor and capital. In addition, firms can adjust utilization u_t by paying a cost $a(u_t)K_{t-1} = \bar{\gamma}_u(e^{\gamma_u(u_t-1)} - 1)K_{t-1}$, where, in the steady state, $a(1) = 0$ and $\bar{\gamma}_u$ is calibrated to obtain $a'(1) = r_{ss}^k$, the steady state rental rate on capital.

Intermediate goods are sold to monopolistically competitive retailers at real price p_t^m. The first order conditions for labor, capital, and utilization are

$$w_t = p_t^m (1-\chi) \frac{Y_t}{H_t}, \qquad r_t^k = p_t^m \chi \frac{Y_t}{K_{t-1}}, \qquad \text{and} \qquad \frac{r_t^k}{u_t} = a'(u_t). \tag{31}$$

2.7 Retailers

Retailers with monopoly power purchase intermediate goods at price p_t^m and set prices for final good varieties. The retailers face Rotemberg adjustment costs for deviating from a target inflation rate $\tilde{\pi}_t$, according to $\frac{1}{2} \cdot \kappa(\frac{\pi_t}{\tilde{\pi}_t} - 1)^2 \cdot Y_t$. In particular, $\tilde{\pi}_t = \bar{\pi}_t^{\iota_p} \pi_{t-1}^{(1-\iota_p)}$ where $\bar{\pi}_t$ represents a time-varying inflation target set by the central bank, and ι_p captures the degree of inflation indexation. As a result, inflation follows a standard Phillips Curve

$$\left(\frac{\pi_t}{\tilde{\pi}_t} - 1\right) \frac{\pi_t}{\tilde{\pi}_t} = \left[P_t^m \frac{\varepsilon_t}{(\varepsilon_t - 1)} - 1\right] \gamma_p + \beta \mathbb{E}_t \Lambda_{t,t+1} \left(\frac{\pi_{t+1}}{\tilde{\pi}_{t+1}} - 1\right) \frac{\pi_{t+1}}{\tilde{\pi}_{t+1}} \frac{Y_{t+1}}{Y_t} \tag{32}$$

where $\gamma_p = (\varepsilon_t - 1)/\kappa$ represents the slope of the Phillips curve.

2.8 Labor agencies and labor unions

Labor agencies demand individual labor varieties from labor unions and aggregate them using a constant elasticity of substitution technology into a composite labor input that is sold to firms.

Optimal demand for labor h_{it} supplied by a labor union to the labor agency is:

$$h_{it} = \left(\frac{w_{it}}{w_t}\right)^{-\varepsilon_t^w} H_t, \tag{33}$$

where ε_t^w is a wage markup which is allowed to vary stochastically and w_t is the real wage paid by firms to purchase the labor composite H_t.

Labor unions choose wages and labor supply for individual labor varieties in order to optimize workers utility. They face Rotemberg costs of adjusting wages $\frac{\kappa^w}{2}(\frac{\pi_t^w}{\tilde{\pi}_t^w} - 1)^2 H_t$, where $\pi_t^w = \frac{w_t}{w_{t-1}}\pi_t$ and $\tilde{\pi}_t^w = \bar{\pi}_t^{\iota_w}\pi_{t-1}^{(1-\iota_w)}$. The parameter ι_w captures the degree of wage indexation to inflation. Labor unions' optimization delivers the wage Phillips curve

$$\frac{\pi_t^w}{\tilde{\pi}_t^w}\left(\frac{\pi_t^w}{\tilde{\pi}_t^w} - 1\right) = \left(\frac{\bar{\varphi}H_t^{\varphi}}{U_{ct}}\frac{\varepsilon_t^w}{\varepsilon_t^w - 1} - W_t\right)\gamma_w + \beta\mathbb{E}_t\Lambda_{t,t+1}\frac{\pi_{t+1}^w}{\tilde{\pi}_{t+1}^w}\left(\frac{\pi_{t+1}^w}{\tilde{\pi}_{t+1}^w} - 1\right)\frac{H_{t+1}}{H_t}, \tag{34}$$

where $\gamma_w = \frac{\varepsilon_t^w - 1}{\kappa^w}$ captures the slope of the wage Phillips curve.

2.9 Capital goods producers

Capital producers sell capital at price Q_t^k and face convex adjustment costs. They solve:

$$\max \mathbb{E}_t \sum_{i=0}^{\infty} \Lambda_{t,t+i}\left[Q_{t+i}I_{t+i} - \mu_t^k I_{t+i}\left(1 - \frac{\gamma_k}{2}\left(\frac{I_{t+i}}{I_{t+i-1}} - 1\right)^2\right)\right]$$

where μ_t^k captures a shock to the marginal efficiency of investment (MEI). Optimality implies the following relation between the price of capital and investment:

$$Q_t^k = \mu_t^k\left(1 + \frac{\gamma_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 + \gamma_k\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right) - \beta\mathbb{E}_t\Lambda_{t+1}\mu_{t+1}^k\gamma_k\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right) \tag{35}$$

2.10 Monetary policy, market clearing, and shocks

Monetary policy sets the interest rate on government bonds according to an inertial Taylor rule that responds to inflation (in deviation from a target) and output growth

$$\log\left(R_{i,t}\right) = (1-\rho_r)\,log(R_{SS}) + \rho_r\log\left(R_{i,t-1}\right) + (1-\rho_r)\left(\kappa_\pi\log\left(\frac{\pi_t}{\bar{\pi}_t}\right) + \kappa_y\log\left(\frac{Y_t}{Y_{t-1}}\right)\right) + \varepsilon_t^m. \tag{36}$$

where ρ_r is a smoothing parameter and ε_t^m is a monetary policy shock.

Market clearing in the goods market requires

$$Y_t - \nu K_{t-1}(1-\gamma_l)\int_0^{\bar{\xi}_t} \xi_t dF_t(\xi_t) - \left(\overline{\psi}\omega_t + \frac{\psi}{2}(\omega_t - \overline{\omega})^2\right) N_t = C_t + I_t + G_t, \tag{37}$$

where the second and third term in the left hand side represent default costs and equity adjustment costs respectively. The parameter ν governs what share of default costs translate into a loss of real resources. Government consumption G_t is subject to exogenous government spending shocks.

The law of motion of aggregate capital is given by

$$K_t = (1-\delta)K_{t-1} + I_{it}. \tag{38}$$

The model features 11 shocks: i) a monetary policy shock ϵ_t^m; ii) a risk shock σ_t; iii) a net worth shock ξ_t^N; iv) a shock to the marginal efficiency of investment μ_t; v) a price markup shock ε_t; vi) a wage markup shock ε_t^w; vii) a TFP shock A_t; viii) a government spending shock G_t; ix) a shock to the inflation target $\bar{\pi}_t$; x) a credit supply shock κ_t; xi) a preference shock ξ_t^c.

As is standard in the literature, we assume that each shock ε_t (expressed in deviation from steady state) has an AR1 representation

$$\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \zeta_t^\varepsilon, \tag{39}$$

where $\zeta_t^\varepsilon \sim N(0, \sigma^\varepsilon)$ is an i.i.d. innovation and the parameter ρ_ε captures the shock persistence. In addition, following Christiano et al. (2014), we introduce a news component for the risk shock, by assuming that

$$\zeta_t^\sigma = \zeta_{0,t}^\sigma + \zeta_{1,t-1}^\sigma + \dots \zeta_{8,t-8}^\sigma, \tag{40}$$

where $\zeta_{0,t}^\sigma \sim N(0, \sigma^\sigma)$ represents the shock innovation realized at time t. The remaining terms are news shocks $\zeta_{1,t-1}^\sigma \dots \zeta_{8,t-8}^\sigma \sim N(0, \sigma_n^\sigma)$, known to the agents up to two years in advance. News shocks have a correlation structure described by the parameter $\rho_{\sigma,n}$. Finally, we assume that the monetary policy shock and the net worth shock have an autocorrelation parameter equal to zero, that is $\rho_N = \rho_m = 0$.

3 Equity adjustment costs and long-term debt

One of our key findings is that equity adjustment costs play a crucial role for how the presence of long-term debt affects the transmission of shocks to the economy. To illustrate this result and to facilitate comparison with existing literature, we study the effect of a negative inflation shock in a version of our model without nominal rigidities ($\kappa = \kappa_w = 0$) and without real costs of default, i.e., $\nu = 0$.[13] This shock is transmitted to the economy by increasing the real burden of corporate debt.[14] We present the response of the economy in *i)* a model without equity adjustment costs and *ii)* a model with infinite equity adjustment costs. In each of these models we compare the response of the economy with long-term debt to a model where all firm debt is short-term.

3.1 Model without equity adjustment costs

We begin by discussing the effects of a shock to inflation in a model without equity adjustment costs, reminiscent of Proposition 1 in Gomes et al. (2016).

When it is costless to adjust equity injections at the margin, i.e., $\psi = \bar{\psi} = 0$, the optimality conditions for the entrepreneur's problem deliver the three following aggregate equations:

$$1 = \frac{Q_t^k}{Q_t^k - Q_t^l \eta_t} \mathbb{E}_t \Lambda_{t,t+1} \int_{\bar{\xi}_{t+1}}^{\infty} \left(\xi_{t+1} - \bar{\xi}_{t+1}\right) R_{t+1}^k dF_t\left(\xi_{t+1}\right), \tag{41}$$

$$\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^l \left(1 - F_t\left(\bar{\xi}_{t+1}^l\right)\right) = \left(1 + \epsilon_{\eta t}\right) - \epsilon_{\eta t} \frac{\left(1-\lambda_l\right)\eta_{t-1}}{\eta_t} \frac{1}{\pi_t} \frac{\left(Q_t^k - Q_t^l \eta_t^l\right)}{\Psi\left(\omega_t\right)\mu_t}, \tag{42}$$

$$K_t = \frac{\omega_t K_{t-1}\left(\tilde{\xi}_t R_t^k Q_{t-1}^k - \frac{\left(c_l + (1-\lambda_l) Q_t^l\right)}{\pi_t}\eta_{t-1}\right) + T_t^e}{Q_t^k - Q_t^l \eta_t}, \tag{43}$$

where equations (41) and (42) are the optimality conditions for equity injections and leverage in equations (12) and (17) repectively, using that $\Psi'(\omega_t) = 1 = \varphi_t = \gamma_t$ when $\psi = 0$. Equation (43) is the aggregate flow of funds constraint for firms, equation (20), using equation (19)

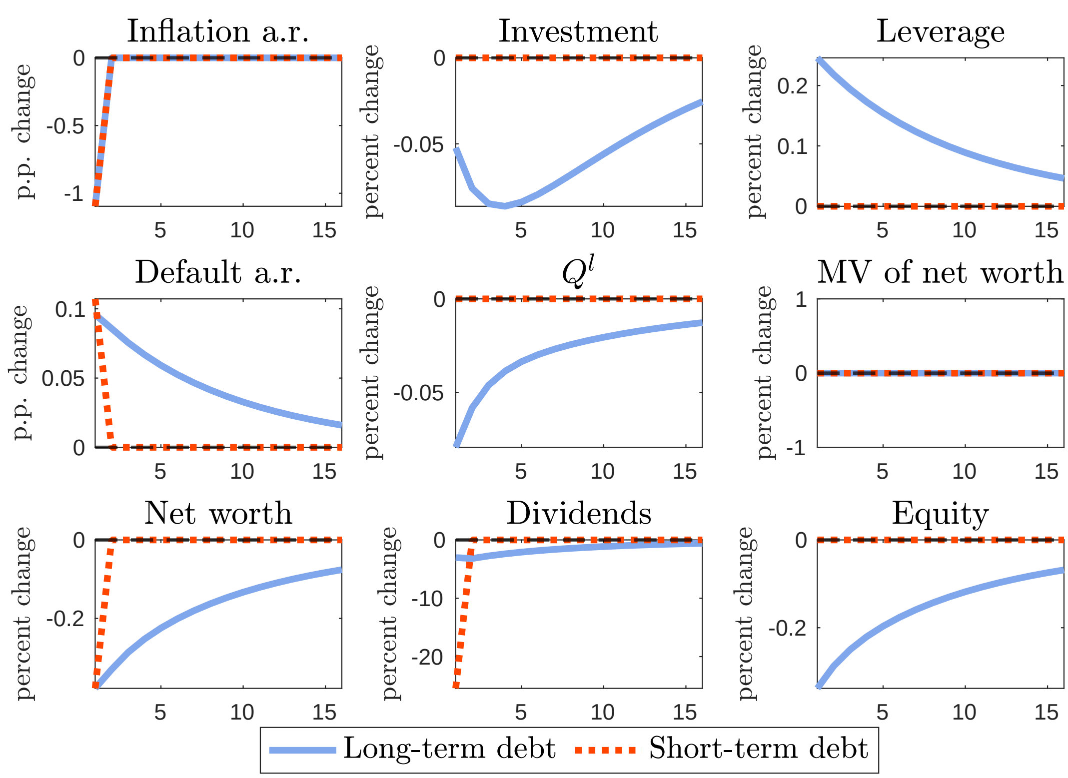
[13] All other parameters are set to the values described in section 4.

[14] In our flexible price economy we implement this shock by removing the monetary policy rule and replacing it with $\pi_t = \exp(\epsilon_t^\pi)$ where ϵ_t^π is an i.i.d. random variable. As shown in Figures D.1 and D.2 in the Appendix, very similar results are obtained if we consider a one-time shock to the value of the outstanding debt, as captured by R_t^l, in our baseline model with nominal rigidities and $\nu = 0$.

to substitute for aggregate equity.[15]

The level of capital demand is determined by equation (41), which requires that the rate of return on capital leaves the entrepreneurs indifferent between paying out dividends and injecting equity. This rate of return will depend on the leverage choice of entrepreneurs, which affects both the total amount of capital per unit of equity, $\frac{Q_t^k}{Q_t^k - Q_t^l \eta_t}$, and the expected default threshold $\bar{\xi}_{t+1}$. Leverage is determined by equation (42), where the last term captures the debt dilution channel associated with long-term debt. Without equity adjustment costs, capital demand is independent of net worth. Entrepreneurs are free to adjust equity injections ω_t to satisfy any given level of capital demand according to (43).

Figure 2: Inflation shock in model with flexible prices: no equity adjustment costs



Note: Figure shows the impulse response to a one time shock to inflation in a simple version of the main model with flexible prices and no real default costs. Inflation and default are shown as annual rates. MV stands for 'marginal value'.

Figure 2 shows the response of the economy to an exogenous one-time decrease in inflation.

[15]In the Appendix we show that when ψ is exactly equal to zero, the first order conditions of the entrepreneur's problem do not select an optimum but a saddle point. Therefore, we can think of the case with no equity adjustment cost as a calibration in which adjustment costs are arbitrarily small. In the Appendix we also show that as ψ goes to zero the derivative of the leverage policy function, η_η, goes to zero as well. See also Ajello, Perez-Orive, and Sző́ke (2023) for additional analysis on the model with no equity adjustment costs.

The blue solid line shows the results of an economy with a debt maturity of seven years, that is $\lambda_l = 1/28$. The dotted red line shows results for an economy with short-term debt, i.e., $\lambda_l = 1$.

In both cases, the decline in inflation causes the real value of entrepreneurs' debt to increase and hence net worth to decline. In the economy with short-term debt, entrepreneurs react to the decline in net worth by reducing net dividend payouts, thus keeping investment, equity, and leverage unaffected. With short-term debt, the choice of leverage (*cf.* the first-order condition in (42) with $\lambda_l = 1$) is not affected by the change in inflation. In contrast, with long-term debt, the increase in the real value of debt gives entrepreneurs an extra incentive to increase leverage, as captured by the last term in equation (42). The larger value of debt makes it more profitable for the entrepreneur to dilute the preexisting creditors by increasing leverage and hence decreasing the price of debt. The increase in leverage causes the expected default rate to go up and the required expected return on capital implied by optimal equity injections in equation (41) to rise. This increase in the returns on capital required by equity investors in the economy with long-term debt captures a *debt overhang channel*, and causes investment to decline in response to a temporary decline in inflation.

3.2 Model with infinite equity adjustment costs

With $\psi = \infty$, the optimality conditions are given by:

$$\omega_t = \bar{\omega}, \tag{44}$$

$$\mathbb{E}_t \Lambda_{t,t+1} R^l_{t+1} \left(1 - F_t\left(\bar{\xi}^l_{t+1}\right)\right) \varphi_{t+1} = \left(1 + \epsilon_{\eta t}\right) \gamma_t - \epsilon_{\eta t} \frac{\left(1 - \lambda_l\right) \eta_{t-1}}{\eta_t} \frac{\left(Q^k_t - Q^l_t \eta^l_t\right)}{\Psi\left(\omega_t\right) \mu_t} \frac{\varphi_t}{\pi_t}, \tag{45}$$

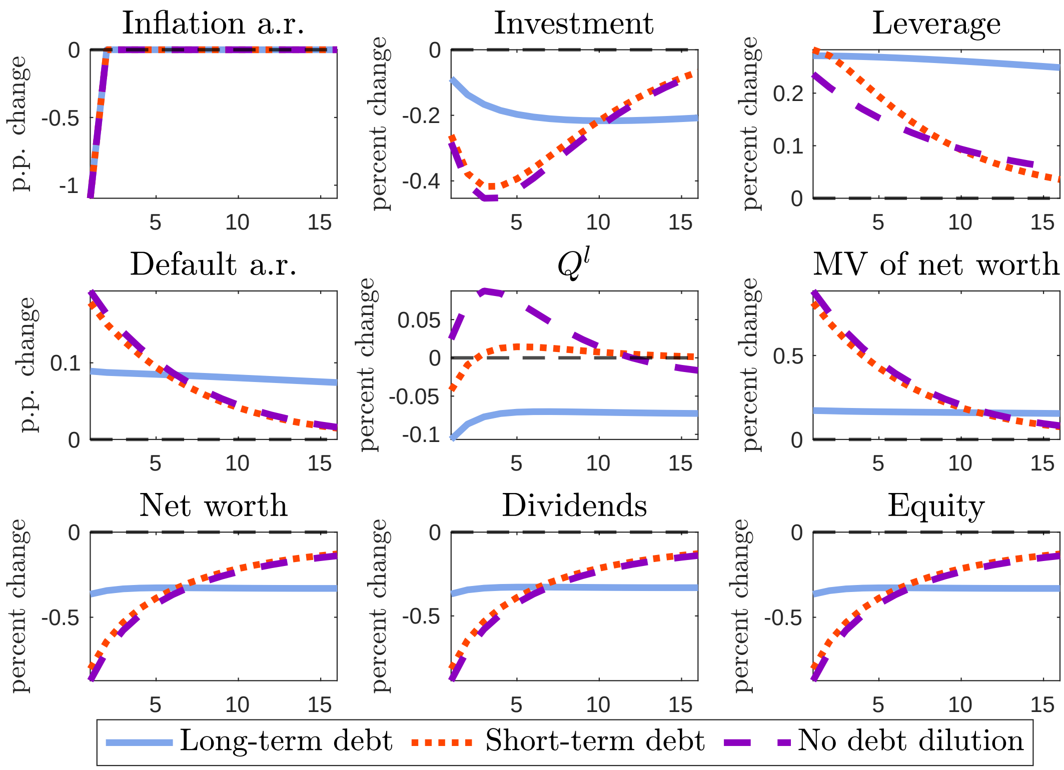
$$K_t = \frac{\bar{\omega} K_{t-1} \left(\tilde{\xi}_t R^k_t Q^k_{t-1} - \frac{\left(c_l + (1-\lambda_l) Q^l_t\right)}{\pi_t} \eta_{t-1} \right) + T^e_t}{Q^k_t - Q^l_t \eta_t}. \tag{46}$$

In this case, entrepreneurs cannot adjust equity injections, as shown in (44), so there is no active debt overhang channel in this economy.[16] Two features will determine the response of aggregate investment and output: the magnitude of the decline in net worth and the degree to which entrepreneurs increase leverage in response to the shock.

Figure 3 shows the response of the economy with infinite equity adjustment costs to the

[16] We set $\bar{\psi} = 0$ for simplicity.

Figure 3: Inflation shock in model with flexible prices: infinite equity adjustment costs



Note: Figure shows the impulse response to a one time shock to inflation in a simple version of the main model with flexible prices and no real default costs. Inflation and default are shown as annual rates. MV stands for 'marginal value'.

same decrease in inflation used in Figure 2. The drop in net worth, caused by the increase in the real value of debt in equation (46), triggers the financial accelerator. Firms borrow more to counter the decline in equity, but entrepreneurs' ability to increase leverage is limited by the associated increase in expected defaults. As a consequence, investment declines, further depressing net worth through a lower Q_t^k. This mechanism, leads to a decline in investment both when debt is long-term (blue solid lines) and when debt is short-term (red dotted lines).[17] Importantly, however, the reduction in investment is now *dampened* in the economy with long-term debt. Two channels explain this result. First, through a *balance sheet channel*, the increase in leverage lowers the price of debt Q_t^l, which, from (46), dampens the fall in net worth. In Figure 3, net worth only falls by about half as much when debt is long-term. Second, because of *debt dilution*, represented by the last term in (45), the elasticity of leverage to a given drop in

[17] The economy with infinite equity adjustment costs and short-term debt is very similar to the framework considered in Bernanke et al. (1999) and Christiano et al. (2014).

net worth is larger with long-term debt. Even with a smaller drop in equity, leverage increases by more in the economy with long-term debt. Taken together, these two channels result in a decline in investment about fifty percent smaller.

An interesting result in our model is that the same debt dilution incentive which, through debt overhang, is responsible for the decline in investment when there are no equity adjustment costs, actually causes a smaller contraction when it is impossible to adjust equity. To highlight this point, we included the purple dashed line in Figure 3, which presents the model responses when we shut down the debt dilution channel.[18] In this case, the impulse response functions are very close to those from the short-term debt model, suggesting that in this experiment the debt dilution effect is responsible for most of the dampening effects of a deflationary shock.[19]

The previous experiments discussed the effects of a stylized shock to inflation. As we discuss below, the dampening influence of long-term debt when equity adjustment costs are large extends to a wide range of shocks, especially financial shocks that affect leverage choices and balance-sheet dynamics. Furthermore, in our framework, the interaction between debt maturity and the behavior of financial variables is important for identifying the drivers of business cycle fluctuations, which we turn to next.

4 Quantitative results

We now present the results of a quantitative version of our model. We briefly discuss the solution method, describe the parameterization and estimation of the model, and present the main insight of the paper, namely how the incorporation of long-term debt and equity adjustment costs affects the response of the economy to shocks.

[18]In particular, we solve the model assuming that the last term in equation (45) is equal to zero. We can think of this alternative specification as a framework in which the outstanding debt is pooled across firms, so that an individual entrepreneur does not internalize how his leverage decision affects the value of outstanding liabilities.

[19]In this exercise the movement in Q_t^l is fairly small resulting in a small balance sheet effect. As we discuss below, this channel can be much stronger when an exogenous shock, like a monetary policy shock or a credit supply shock, directly lowers the price of outstanding debt.

4.1 Solution method

Defaultable long-term debt complicates the solution of the model by requiring the computation of the derivative of entrepreneurs' individual leverage policy function, $\eta_{\eta,t}(\eta_{t-1})$, which appears in (26). To overcome the associated computational challenge, we use an algorithm combining global methods for the solution of the model's steady state with perturbation methods to obtain impulse response functions.[20] In the first step of the algorithm, we use global methods to compute the policy function for leverage as a function of η_{t-1} and of a set of aggregate prices and quantities in steady state Γ^{SS}, that is $\eta_t(\eta_{t-1}, \Gamma^{SS})$ and its derivative, $\eta_{\eta,t} = \frac{\partial \eta_t}{\partial \eta_{t-1}} = g(\eta_{t-1}, \Gamma^{SS})$, in steady state. This result is achieved by an iterative procedure which solves for the model's steady state and for $\eta_{\eta,ss}$ simultaneously. Figure 4 shows the leverage policy function and the price of long-term debt on a grid of values for past individual leverage at the steady state of our model, as obtained from the first step of our algorithm. Leverage is upward sloping, due to the debt dilution incentive, and the derivative of this policy function at the steady state value of η is about 0.9. The right panel shows that Q^l is decreasing in leverage, as higher debt increases the probability of default.[21]

In the second step of the algorithm, we approximate the derivative function, $g(\eta_{t-1}, \Gamma_t)$, with a linear function $\hat{g}(\eta_{t-1}, \Gamma_t)$ in the neighborhood of the steady state. This function can then be used as an additional equation of the model. As a result, standard perturbation techniques can be used to simulate the model dynamics. In addition, once we have calibrated the parameters determining the steady state values of entrepreneurial variables (such as equity adjustment costs, leverage, default rates, and spreads), this procedure allows us to use standard Bayesian techniques to estimate the model, as we discuss below.

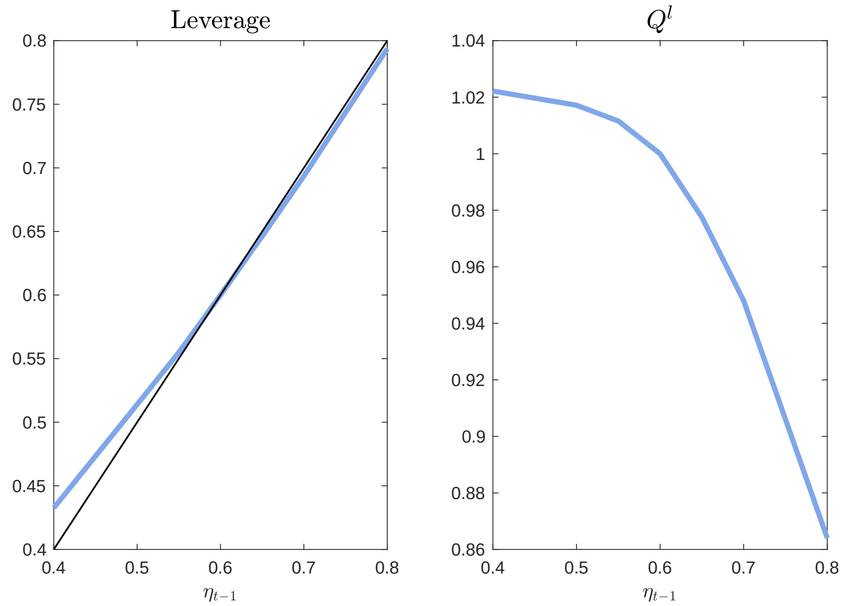
4.2 Parameters

We divide the model parameters into three groups. The first group is calibrated to steady state moments, the equity adjustment cost parameter ψ is estimated using firm-level balance sheet data, and the remaining parameters are estimated using Bayesian methods. This approach guarantees that the approximation of the derivative of the entrepreneur's policy function, $\hat{g}(\hat{\eta}_{t-1}, \hat{\Gamma}_t)$,

[20] A similar algorithm was also used in Ferrante (2019). The Appendix describes the method in detail.

[21] In addition, we check numerically that the value function of the entrepreneur is concave in η in the neighborhood of the steady state.

Figure 4: Policy functions for the individual entrepreneur



Notes: The left and right panel report the individual leverage policy function and the price of debt, Q^l , computed over a grid of past leverage, around the steady state of the model. The black line in the left panel represents the 45 degree line.

which is computed using global methods, is not affected by the values of the third group of parameters, making estimation feasible.

Calibrated parameters Table 1 reports the calibrated parameters, which are mostly set to standard values from the literature. For household preferences, we assume log utility and a discount factor that implies a steady state real interest rate of 2 percent. We choose an elasticity of substitution across goods- and labor varieties of six. Government spending is set to 20 percent of output in the steady state.[22]

The parameters pertaining to entrepreneurs are specific to our model. We set λ_l to obtain a debt duration of seven years, a value in line with the mean bond duration reported by Gilchrist and Zakrajšek (2012). The target dividend payout ratio is set to $1-\bar{\omega} = .015$ to obtain a steady state dividend/output ratio of around four percent, in line with the data. The parameters τ^e, σ_l, γ_l are determined jointly. The targets were *i)* a leverage ratio of 0.6, *ii)* an annual default rate of three percent, *iii)* a bonds spread of 100 basis points annualized. The target for leverage

[22]Furthermore, in order to account for the downward trend in inflation in the early part of the sample, we follow Christiano et al. (2014) and calibrate the parameters governing the inflation target shock to $\rho_{\bar{pi}} = 0.975$ and $\sigma_{\bar{pi}} = 0.0001$.

Table 1: Calibrated parameters

Parameter	Value	Description	Target/Source
Households			
β	0.995	Discount factor	2% Real rate
σ	1	IES	Standard
φ	1	Inverse Frisch El.	Standard
Intermediate Production			
χ	0.4	Capital share	Standard
ε_p	6	Ela. of subst. goods	Standard
ε_w	6	Ela. of subst. labor	Standard
Investment Production			
δ_k	0.025	Depreciation rate	Standard
Government			
G/Y	0.2	Gov. exp. to GDP	Standard
Entrepreneurs			
λ_l	0.0357	Debt duration	Debt maturity: 7 yrs.
$1-\bar{\omega}$	0.015	Dividend payout	4% Dividend/GDP
τ^e	0.01	Transfer new ent.	$\eta^l = 0.6$
σ_l	0.26	St. Dev. idiosyncratic risk	3% Default rate
γ_l	0.79	Recovery rate	$R^l - R = 100\,\text{bp}$
$\bar{\psi}$	0.0045	Linear equity iss. cost	$\omega_{ss} = \bar{\omega}$
ν	1	Resource cost of default	Literature
ψ	10.86	Quadratic equity iss. cost	Compustat

implies a ratio of capital to net worth $K/N = 2.5$ in line with the evidence in Kalemli-Ozcan, Sorensen, and Yesiltas (2012). The default rate is close to the value used in Bernanke et al. (1999). The spread is calibrated to the average value of the difference between Moody's BAA corporate yield and the AAA corporate yield between 1986 and 2019. Finally we normalize $\bar{\psi}$ so that in steady state there are no quadratic costs of issuing equity, i.e. $\omega_{ss} = \bar{\omega}$.[23]

Estimation of equity adjustment costs The last row of Table 1 shows our estimated value of ψ, the parameter governing the quadratic equity adjustment costs. To a first order, this parameter only affects the optimality condition for entrepreneurs' equity injections. As we described in Section 3, this parameter plays a key role in our model because it determines how long-term debt affects the response of the economy to shocks. To estimate ψ we use the

[23] We assume that default costs entail a resource loss ($\nu = 1$) as is standard in the literature.

optimality condition for equity injections, (9), which we reprint here for convenience:

$$1 = \frac{\Psi'\left(\omega_t\right)}{Q_t^k - Q_t^l \eta_t} \cdot \mathbb{E}_t \Lambda_{t,t+1} \int_{\bar{\xi}_{t+1}}^{\infty} v_{t+1}\left(\eta_t, \xi_{t+1}\right) dF_t\left(\xi_{t+1}\right)$$

We define the return on equity injections as

$$R_{t+1}^x = \frac{\int_{\bar{\xi}_{t+1}}^{\infty} v_{t+1}\left(\eta_t, \xi_{t+1}\right) dF_t\left(\xi_{t+1}\right)}{Q_t^k - Q_t^l \eta_t}$$

and use the functional form for equity adjustment costs, $\Psi'(\omega) = 1 - \bar{\psi} - \psi(\omega_t - \bar{\omega})$ to write

$$\mathbb{E}_t \Lambda_{t,t+1} R_{t+1}^x = \frac{1}{1 - \bar{\psi} - \psi(\omega_t - \bar{\omega})}, \tag{47}$$

which we linearize to obtain

$$\mathbb{E}_t r_{t+1}^x - r_{t+1} = \frac{\psi}{1 - \bar{\psi}} d\omega_t. \tag{48}$$

Here r_{t+1}^x and r_{t+1} denote percent deviation of R_{t+1}^x and the real risk-free rate from their steady state values. We then use firm-level balance sheet data from Compustat to estimate this relationship in the data.

Our sample consists of 19,341 US non-financial firms between 1985q1 and 2023q1.[24] In the data, we define the return on equity as firm i's time $t+1$ cum-dividend market value of equity divided by its time t cum-dividend book value of equity, i.e., $R_{i,t+1}^x = \frac{V_{i,t+1}}{X_{i,t}}$. The end-of-quarter market value of equity, $V_{i,t+1}$, is measured using Compustat items `cshoq` and `prccq`. The book value of equity, $X_{i,t}$, is given by `ceqq`. Net dividend payouts are given by dividends (`dvy`), minus net repurchases (`sstkyq - prstkcyq`). This lets us define ω_{it} as one minus the fraction of net payouts to shareholders over the cum-dividend book value of equity. We run the following regression:

$$\frac{R_{i,t+1}^x}{1 + r_{t+1}} = \psi \omega_{i,t} + \delta_i + \delta_t + \nu_{it} \tag{49}$$

Here r_{t+1} is the quarterly realized real rate and δ_i and δ_t are firm- and time fixed effects.[25]

This approach produces a value of $\psi = 10.86$ with a standard error of 0.13. The result is robust to the inclusion of the fixed effects. Our estimate of ψ implies significant equity adjustment costs, resulting in model dynamics close to those of a calibration with $\psi = \infty$.[26]

[24]Details are presented in Appendix C.
[25]We omit $\bar{\psi}$ which, being a very small number, does not affect the results in any meaningful way.
[26]Appendix C presents additional details on the robustness of our estimation. Appendix D compares the

Estimated parameters We estimate the remaining parameters of the model using Bayesian methods. We use quarterly observations on ten variables between 1985Q4 and 2019Q4. Those include seven standard macroeconomic variables (GDP, investment, consumption, hours worked, inflation, real wages, and the Federal Funds Rate) in addition to three financial variables. The latter include the spread between BAA-rated corporate bonds and the ten-year US government bond rate, total credit to non-financial firms from the U.S. Financial Accounts, and equity valuations from the Wilshire 5000 index. The model equivalents to the financial variables are the long-term debt spread, $\mathbb{E}_t \frac{\sum_{i=1}^{28}(R^l_{t+i} - R^d_{t+i})}{7}$, total credit $Q^l_t L_t$, and total equity X_t. We take log differences for GDP, investment, consumption, credit, equity and real wages, and we remove the sample mean from all observable variables.[27]

Table 2 reports the priors and posteriors for the estimated parameters. The priors are calibrated as in Christiano et al. (2014). The posterior standard deviations are for the most part significantly smaller than the standard deviation of the prior distributions, suggesting that the data are informative about the estimated parameters. The top panel of Table 2 reports the values for the economic parameters, which are within the range of estimates in the literature. The second panel reports the results for the shock autocorrelation parameters. Most of the shocks are quite persistent, with the exception of the wage markup shock. The standard deviations of the macroeconomic shocks are reported in the bottom panel of Table 2. One thing to notice is that our estimation implies that the standard deviation of the anticipated component of the risk shock $\sigma_{\sigma,n}$ is larger than the one on the unanticipated component σ_σ. As a result, most of the fluctuations in σ_t are due to anticipated shocks.

impulse responses of our baseline economy to those from a calibration with $\psi = \infty$.

[27] Real consumption is the sum of nondurable goods and services. Real investment is obtained as the sum of gross private domestic investment plus durable goods purchases. Aggregate hours are an index of nonfarm business hours for all persons. These variables are converted into real, per-capita, terms by dividing them by the population over 16 and by the GDP implicit price deflator. Inflation is measured as the change in the GDP price deflator. Real wages are given by the hourly compensation of all nonfarm employees divided by the GDP deflator. For the Federal Funds Rate, we use the shadow rate from Wu and Xia (2016). Credit is measured as total liabilities for nonfarm, non-financial corporate business from the Financial Accounts. As equity, measured by the Wilshire 5000 index, it is converted to real per-capita terms as above. Data are obtained through Haver Analytics.

Table 2: Estimated parameters

Parameter	Description	Prior Dist.	Prior Mean	Prior S.D.	Posterior Mode	Posterior S.D.
Economic parameters						
s_p	Slope price Phillips curve	Beta	0.05	0.01	0.0415	0.0067
s_w	Slope wage Phillips curve	Beta	0.05	0.01	0.0181	0.0037
ι_p	Price indexing	Beta	0.50	0.15	0.9000	0.0400
ι_w	Wage indexing	Beta	0.50	0.15	0.4400	0.1500
κ_π	Policy weight inflation	Normal	1.50	0.25	2.5300	0.1400
κ_y	Policy weight growth	Normal	0.25	0.10	0.6400	0.0700
ρ_i	Policy inertia	Beta	0.75	0.10	0.8500	0.0100
h	Habit	Beta	0.50	0.10	0.8300	0.0200
γ_I	Investment costs	Normal	5.00	3.00	1.2000	0.1500
γ_u	Utilization costs	Normal	2.00	1.00	4.7000	0.7000
Autocorrelation parameters						
ρ_κ	Credit supply	Beta	0.50	0.20	0.8400	0.0120
ρ_σ	Risk	Beta	0.50	0.20	0.9800	0.0080
$\rho_{\sigma,n}$	Risk news	Beta	0.50	0.20	0.5800	0.0080
ρ_I	M.E.I.	Beta	0.50	0.20	0.9900	0.0020
ρ_ε	Price markup	Beta	0.50	0.20	0.7900	0.0370
ρ_z	TFP	Beta	0.50	0.20	0.9700	0.0180
ρ_g	Government	Beta	0.50	0.20	0.9400	0.0150
ρ_c	Consumption	Beta	0.50	0.20	0.8000	0.0400
ρ_{ε_w}	Wage markup	Beta	0.50	0.20	0.1100	0.0650
Standard deviation parameters						
σ_κ	Credit supply	IG2	0.002	0.0048	0.0070	0.0010
σ_σ	Risk	IG2	0.002	0.0033	0.0001	0.0030
$\sigma_{\sigma,n}$	Risk news	IG2	0.002	0.0160	0.0280	0.0030
σ_I	M.E.I.	IG2	0.002	0.0033	0.0210	0.0010
σ_ε	Price markup	IG2	0.002	0.0033	0.0010	0.0001
σ_z	TFP	IG2	0.002	0.0033	0.0046	0.0003
σ_g	Government	IG2	0.002	0.0033	0.0170	0.0010
σ_c	Consumption	IG2	0.002	0.0033	0.0260	0.0032
σ_{ε_w}	Wage markup	IG2	0.001	0.0033	0.0090	0.0001
σ_N	Equity shock	IG2	0.002	0.0033	0.0280	0.0017
σ_R	Monetary policy	IG2	0.583	0.8250	0.4600	0.0355

4.3 Credit supply shocks and business cycle fluctuation

The main takeaway from our estimated model with long-term debt and costly equity issuance is that the credit supply shock is the main driver of business cycle fluctuations. We present this result in Table 3, which reports the unconditional variance decomposition of key macroeconomic and financial variables. The table is divided into four panels. The first panel reports the variance decomposition for our estimated baseline model, which includes long-term debt and costly equity issuance. It shows that the credit supply shock is the main driver of output fluctuations, accounting for roughly half of the business cycle variance of GDP growth and investment growth. The credit supply shock also explains a large part of the fluctuations in financial variables, accounting for 47 percent of the variance of the Federal Funds Rate, 21 percent of the variance of credit growth and 15 percent of the variance of equity fluctuations.[28]

Figure 5 complements the results in the first panel of Table 3 by conducting an in-sample exercise that compares the behavior of real and financial variables from the data (purple solid line) with a model simulation, obtained by feeding only the estimated credit supply shocks into the model (blue starred line). The top left panel shows that the credit supply shock does well at reproducing the time series for output growth over our estimation sample. In particular, the large contraction in output during the 2008 financial crisis is associated with a large negative credit supply shock. The credit supply shock also tracks the behavior of hours and the policy rate well. The remaining panels in Figure 5 show that the shock generates financial time series in line with the data, especially for credit growth and credit spreads.

Finally, we show that our estimate for the equity adjustment cost parameter ψ, obtained from cross-sectional data, is consistent with the time series behavior of dividends in the data. The model-implied dividends to GDP ratio, which is compared to the data in Figure 6, is obtained by feeding the estimated credit supply shocks into the model. Although this variable was not used in the estimation, the model produces a time series that is close to its empirical counterpart. In particular, the model captures the procyclical nature of dividend payouts. This evidence suggests that our estimate for ψ results in a realistic time series behavior for aggregate dividends in the model.

The principal reason why the credit supply shock is identified as the driving force of the business cycle in our model is that it delivers impulse responses with the right co-movement between real and financial variables. Figure 7 illustrates this result by reporting the impulse responses to a credit supply shock, shown as the blue solid lines. For comparison, we also

[28] The credit supply shock accounts for a small share of the unconditional variance of spreads, while the bulk of this variable is explained by the risk shock. This result is likely due to the fact that the additional news component of the risk shock, together with its higher persistence, allows this shock to affect the forward-looking spread variable in our model more directly, that is $\mathbb{E}_t \sum_{i=1}^{28} (R^l_{t+i} - R^d_{t+i})$.

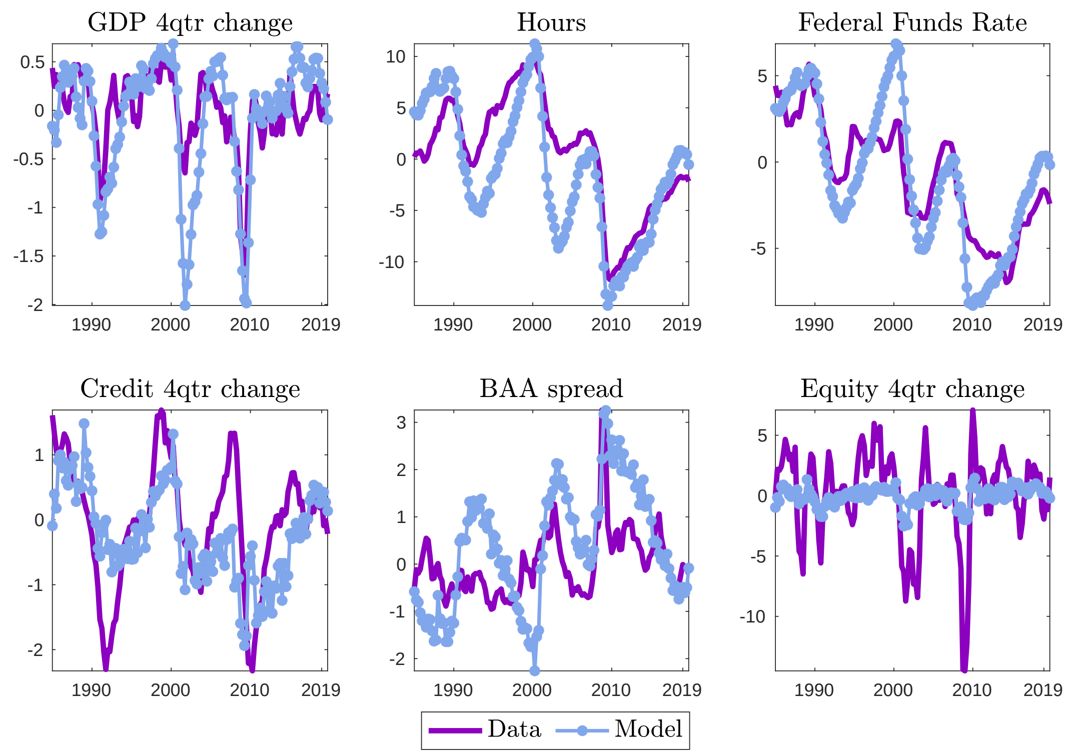
Table 3: Variance decomposition

	Credit Supply	Risk	M.E.I.	NW	Markup	Gov.	M.P.	TFP
	Baseline model							
GDP	44	4	6	5	19	8	4	4
Investment	52	4	15	5	14	0	3	2
Consumption	6	4	23	1	2	1	0	4
Spread	6	63	27	3	0	0	0	0
Equity	15	4	13	64	1	0	2	0
Credit	21	10	34	31	1	0	1	0
Inflation	37	7	10	1	21	1	5	5
FFR	47	11	29	2	1	1	2	1
Hours	26	9	37	1	16	2	2	2
Wages	1	1	3	0	10	0	0	3
	Model with long-term debt – no equity in estimation							
GDP	8	1	25	2	19	18	5	6
Investment	28	2	38	3	16	0	5	3
Consumption	10	0	0	0	2	2	1	6
Spread	47	22	0	12	0	2	0	2
Equity	13	1	0	70	2	1	8	0
Credit	16	4	4	45	7	2	5	1
Inflation	19	1	4	1	33	2	8	6
FFR	42	2	7	2	4	4	8	2
Hours	22	2	9	1	26	5	3	4
Wages	1	0	0	0	10	0	0	3
	Model with short-term debt							
GDP	5	24	4	9	24	10	5	8
Investment	11	35	13	12	17	0	4	4
Consumption	6	4	15	1	3	1	0	7
Spread	99	1	0	0	0	0	0	0
Equity	5	11	29	48	3	0	5	0
Credit	2	18	9	66	2	0	1	1
Inflation	7	22	18	2	18	1	5	8
FFR	14	28	33	3	2	1	4	2
Hours	10	17	20	2	26	2	2	4
Wages	1	2	2	0	11	0	0	4
	Model with long-term debt and $\psi = 0$							
GDP	9	54	5	3	17	2	6	3
Investment	9	53	9	4	17	0	6	1
Consumption	4	15	24	1	22	1	1	8
Spread	9	21	25	2	39	0	1	3
Equity	11	4	11	50	17	0	4	2
Credit	12	11	44	13	16	1	1	1
Inflation	7	28	26	1	28	0	3	4
FFR	6	32	24	2	30	0	2	2
Hours	2	15	14	1	60	1	2	4
Wages	2	13	7	0	19	0	1	3

Note: This table reports the variance decomposition of the observables used in the estimation. The decomposition is computed at the posterior mode for the estimated parameters. The column labeled "Risk" combines the contribution of the unanticipated and anticipated risk shocks. "M.E.I." stands for marginal efficiency of investment.

include two other shocks the literature has identified as important drivers of economic activity: a risk shock (green dotted line) and a shock to the marginal efficiency of investment (yellow dashed line). The credit supply shock reduces lenders' appetite for corporate debt and increases credit spreads by about 40 basis points. With sizeable equity issuance costs, entrepreneurs do

Figure 5: The role of the credit supply shock

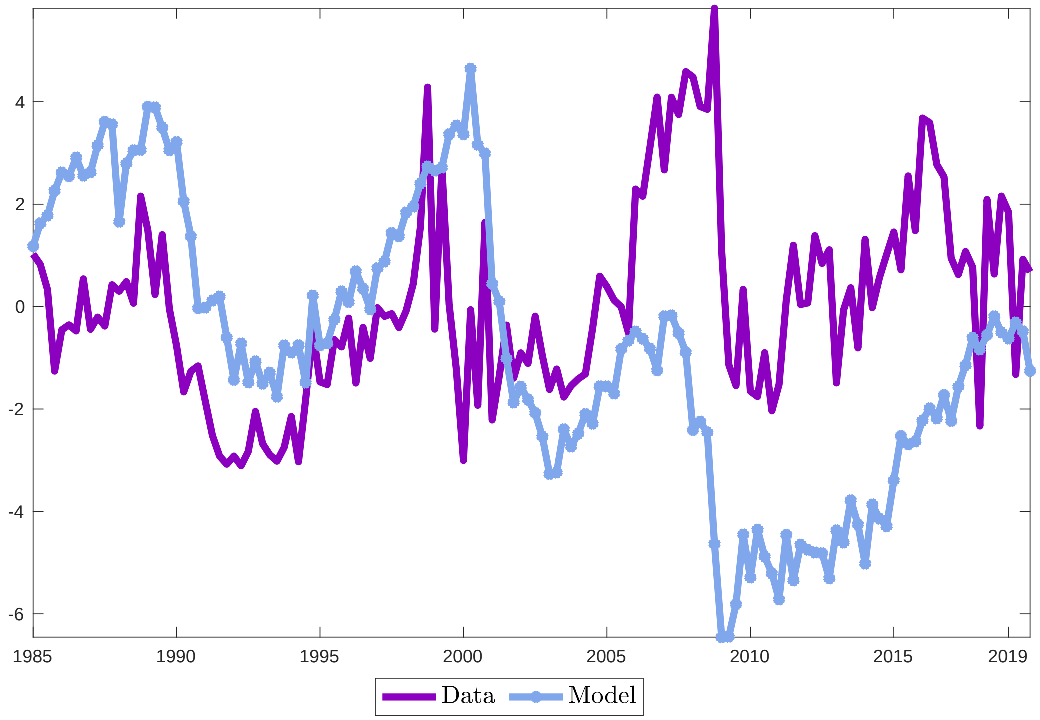


Note: Figure compares the behavior of selected variables in the data (purple lines) with the model-implied behavior attributed to the filtered credit supply shocks (blue lines). Variables are reported in deviation from their mean. Data source: Haver Analytics.

not make up for the decline in debt with external equity. As a result, firms cut investment, putting downward pressure on asset prices and causing equity to decline. This mechanism implies a standard financial accelerator channel as in Bernanke et al. (1999), which results in a large contraction in investment, output, and consumption. The credit supply shock delivers a co-movement between real and financial variables consistent with the data.

In contrast, as shown in the bottom row of Figure 7, the risk shock and the shock to the marginal efficiency of investment, both induce a countercyclical movement of equity, which is at odds with the data. The shock to the marginal efficiency of investment is an exogenous contraction in the supply of investment goods that causes investment and output to decline and the price of capital to increase. As a result, firms' asset values - and with it equity - rise during the downturn. This counterfactual behavior of equity, following a shock to the marginal efficiency of investment, is discussed by Christiano et al. (2014) as the main reason in favor of risk shocks in a model with short-term debt. With long-term debt, however, the risk shock

Figure 6: The role of the credit supply shock: Dividends/GDP



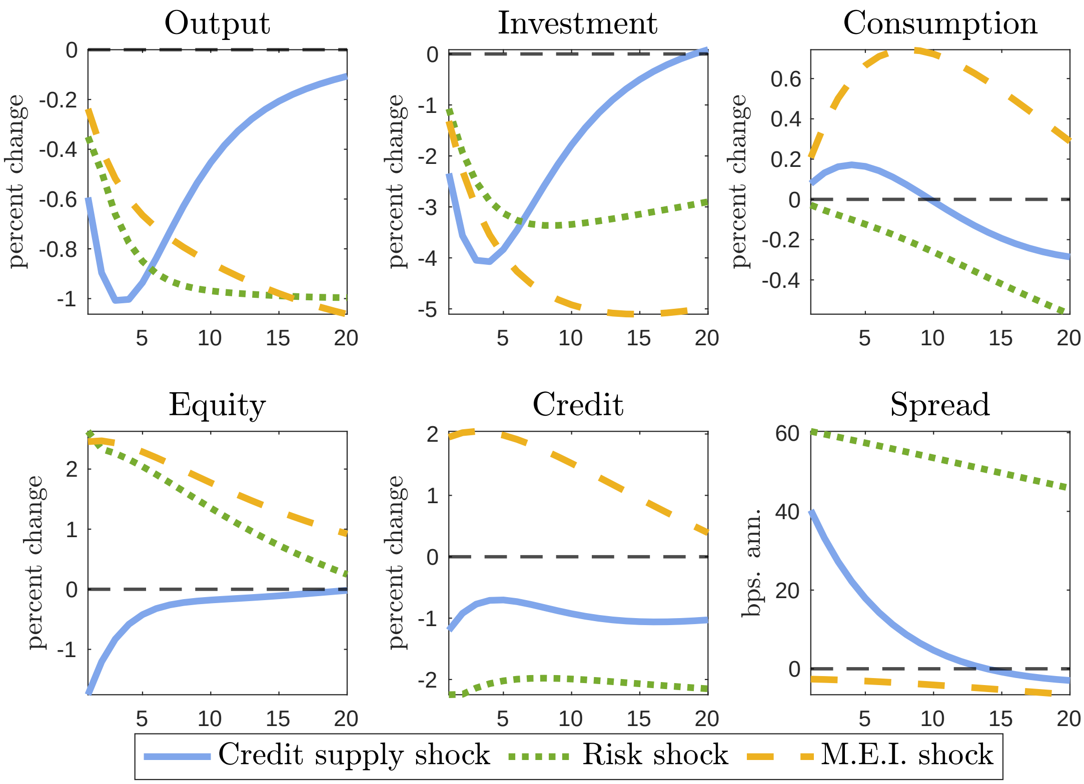
Note: Figure compares the behavior of Dividends/GDP in the data (purple lines) with the model-implied behavior attributed to the filtered credit supply shocks (blue lines). Variables are reported in deviation from their mean. Dividends are measured as in Jermann and Quadrini (2012), as dividends and share repurchases minus equity issues of nonfinancial corporate businesses, minus net proprietor's investment in noncorporate businesses. Data source: US Financial Accounts.

also leads to a counter-cyclical response of equity. When $\lambda_l < 1$, the risk shock causes a large increase in firms' debt dilution incentive by making the price of debt, Q_t^l, more sensitive to leverage. This is because with a larger variance of idiosyncratic shocks, the density around the default threshold increases and hence an increase in leverage causes a larger increase in defaults and a larger drop in the price of debt.[29] Figure 8 illustrates this point by showing the responses of the debt dilution incentive (the last term in (17)), together with the prices of debt and assets after a credit supply shock (blue solid line) and a risk shock (black dashed line). The large increase in the debt dilution incentive following a risk shock causes a larger drop in the value of debt and a smaller drop in the price of capital compared to the responses to a credit supply shock. As a result, equity increases after a risk shock.[30]

[29]See equation (26).

[30]Figure D.3 in the Appendix shows the impulse response to a net worth shock in our model. As suggested by Christiano et al. (2014), this shock generates a countercyclical response of credit.

Figure 7: Credit supply shock, risk shock, and MEI shock

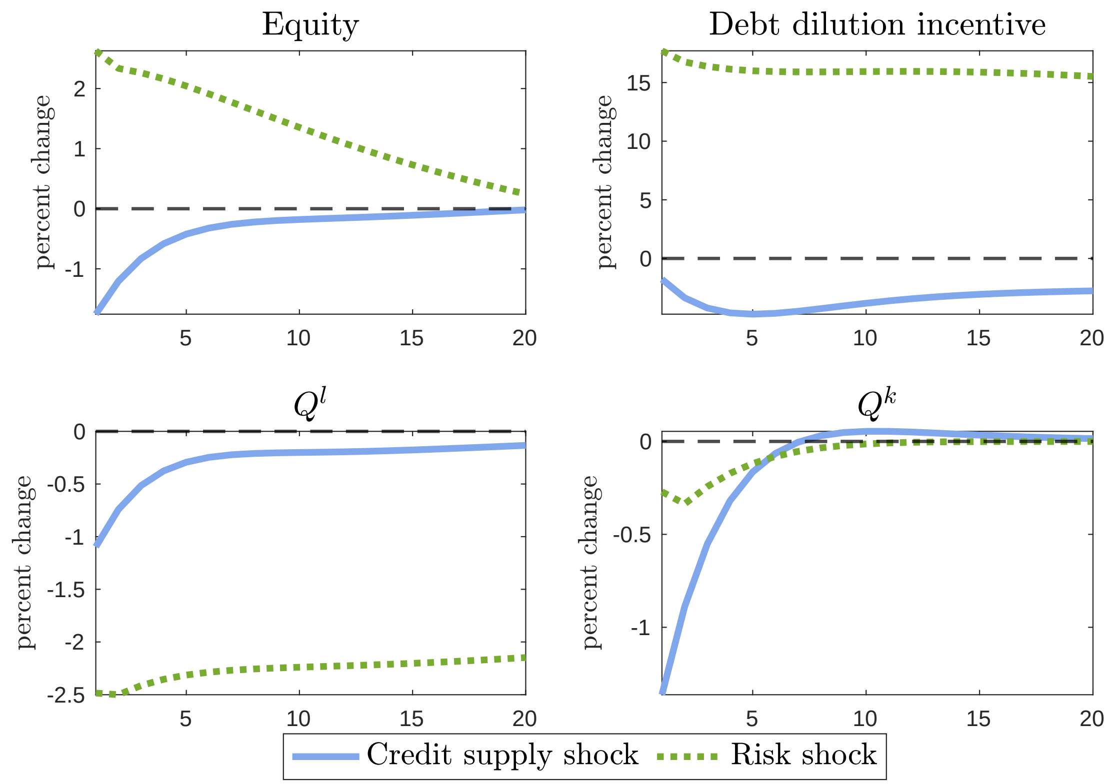


Note: Figure shows the impulse response to a credit supply shock (blue solid line), a risk shock (green dotted line), and a shock to the marginal efficiency of investment (yellow dashed line) in our estimated model.

To highlight the importance of including equity prices among the variables used in the estimation, the second panel of Table 3 shows that if we remove equity as an observable, the main driver of the business cycle becomes the shock to the marginal efficiency of investment as in Justiniano et al. (2010).

The role of long-term debt Assumptions on corporate debt maturity have important implications for the inference of which shocks drive the business cycle. Restricting the model to short-term debt shifts the balance from credit supply shocks to risk shocks, as we show in the third panel of Table 3. This panel reports the variance decomposition obtained when re-estimating our model under the assumption that debt is short-term, i.e., $\lambda_l = 1$. In this case, the risk shock becomes the main driver of fluctuations in output and investment. This result is in line with Christiano et al. (2014) who find that, in a model with short-term debt,

Figure 8: Equity response to credit shock and risk shock



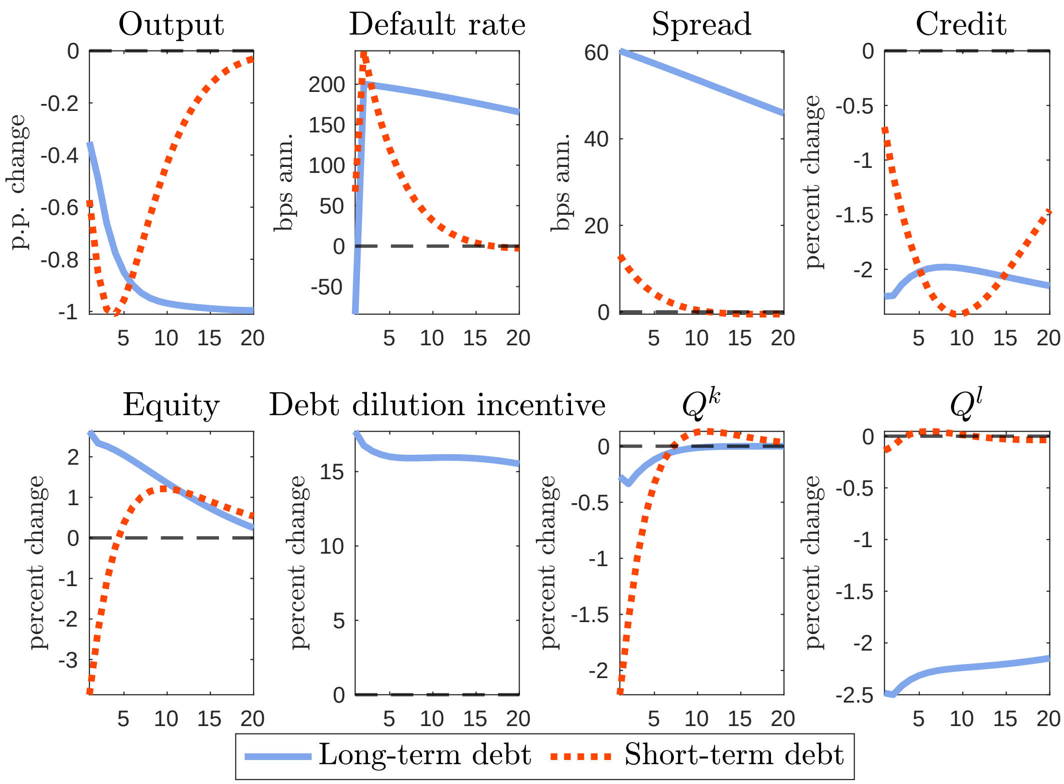
Note: Figure reports the impulse response to a credit supply shock (blue line) and to a risk shock (green dotted line) in our estimated model. The debt dilution incentive variable is the last term in equation (17).

fluctuations in entrepreneurial risk, σ_t, are the most important shock for the business cycle.[31]

The reason for this result is that in a model with short-term debt an increase in risk induces the right co-movement between macro and financial variables. This point is illustrated in Figure 9, which shows the economy's response to a risk shock in our baseline (blue solid line) and in the estimated model with short-term debt (red dotted line). The risk shock increases the dispersion of future idiosyncratic entrepreneurial shocks, implying persistently higher default rates and causing spreads to rise and credit to decline. When debt is short-term, the debt dilution incentive is absent and there is no balance sheet effect from fluctuations in the price of debt. As a result, an increase in risk causes a larger drop in asset prices than in the case with

[31]When setting $\lambda_l = 1$, our model is very similar to that used in Christiano et al. (2014), other than the following key differences in the modeling assumptions and the estimation strategy: *i)* Christiano et al. (2014) use the contractual framework of Bernanke et al. (1999) which assumes that entrepreneurs bear all the risk from aggregate shocks, whereas we let lenders face credit risk; *ii)* we use a longer estimation sample and a slightly different set of shocks; *iii)* in the estimation, Christiano et al. (2014) match the BAA spread, a measure based on long-term securities, with the realized one-period spread in their model; *iv)* in our baseline calibration $\psi = 10.86$, whereas Christiano et al. (2014), following Bernanke et al. (1999), assume $\psi = \infty$.

Figure 9: Risk shock in baseline model and in model with short-term debt

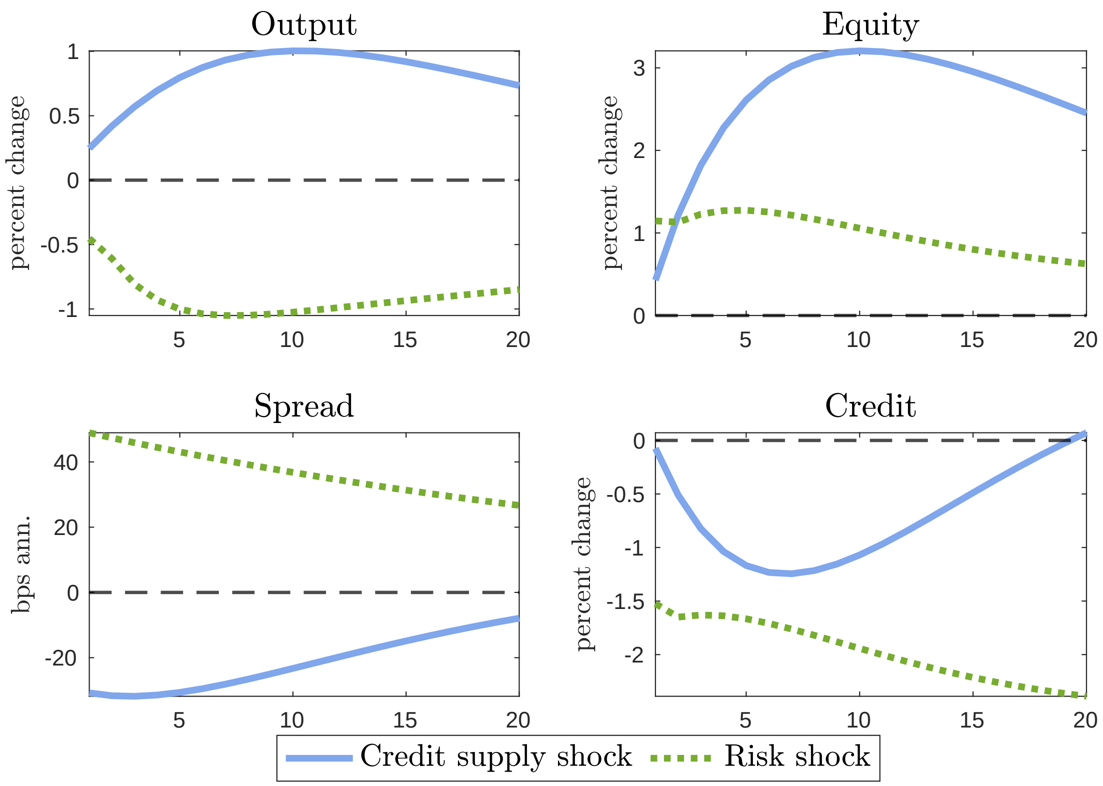


Note: Figure reports the impulse responses to a risk shock in our estimated baseline model with long-term debt (blue line) and in the estimated model with short-term debt (red dashed line). Shocks are normalized to deliver a one percent decline in output.

long-term debt, implying a decline in firms' equity. The risk shock thus induces a pro-cyclical response of equity prices in a model with short-term debt, in line with the data.

The role of equity adjustment costs The second key ingredient of our estimated model are the equity adjustment costs. The bottom panel of Table 3 shows that if we estimate the model with $\psi = 0$, we obtain a much smaller role for the credit supply shock, whereas the risk shock is again the main driver of fluctuations in output and investment. Figure 10 shows the impulse response to a credit shock (blue solid line) and a risk shock (green dotted line) in the estimated model without equity adjustment costs. Notice that the exogenous increase in the interest rate on firms' liabilities, caused by the credit supply shock, now results in an *increase* in output. When equity adjustments are costless, firms respond to a credit supply shock by substituting debt issuance with equity issuance. As a result, leverage and default rates fall, weakening the debt overhang effect. This leads to investment and output increasing, while credit spreads tighten and total debt declines. This counterfactual countercyclical behavior of debt in response to credit supply shocks leads the estimation to favor the risk shock in this case. However, the marginal data density of the model with $\psi = 0$ is 4689, more than 300 points below the marginal data density in our baseline model of 5031.

Figure 10: Credit shock and risk shock in model without equity adjustment costs



Note: Figure reports the impulse response to a credit supply shock (blue line) and to a risk shock (green dotted line) in a model with no equity adjustment costs ($\psi = 0$).

While without equity adjustment costs the model dynamics change substantially, we show that our baseline calibration behaves similarly to a model with infinite adjustment costs. Figure D.4 in the Appendix shows that the impulse responses in our baseline model and in a model with $\psi = \infty$ are very close. In addition, Table A2 reports the variance decomposition when we re-estimate the model with $\psi = \infty$ and shows that the results are very similar to those obtained in our baseline calibration with $\psi = 10.86$ (top panel of Table 3).[32] These results suggest that our estimated value for ψ results in substantial equity frictions.

Overall, the results in Table 3 suggest that to capture the importance of credit supply shocks

[32]However, the marginal data density with $\psi = \infty$ is 5017, lower than in our baseline specification.

in an estimated DSGE model with financial frictions, three elements are necessary: *i)* using financial variables as observables, *ii)* the presence of long-term debt, and *iii)* having equity adjustment costs consistent with the data.

4.4 Debt maturity and macroeconomic shocks

In Section 3, we studied how corporate debt maturity affects the response to a stylized shock to inflation in a simplified version of our model. Figure 11 shows how debt maturity affects the transmission of standard macroeconomic shocks by comparing the impulse responses to five different shocks in the baseline model with a debt maturity of seven years (blue line) and in a model with an identical parameterization but with short-term debt (red dashed line).

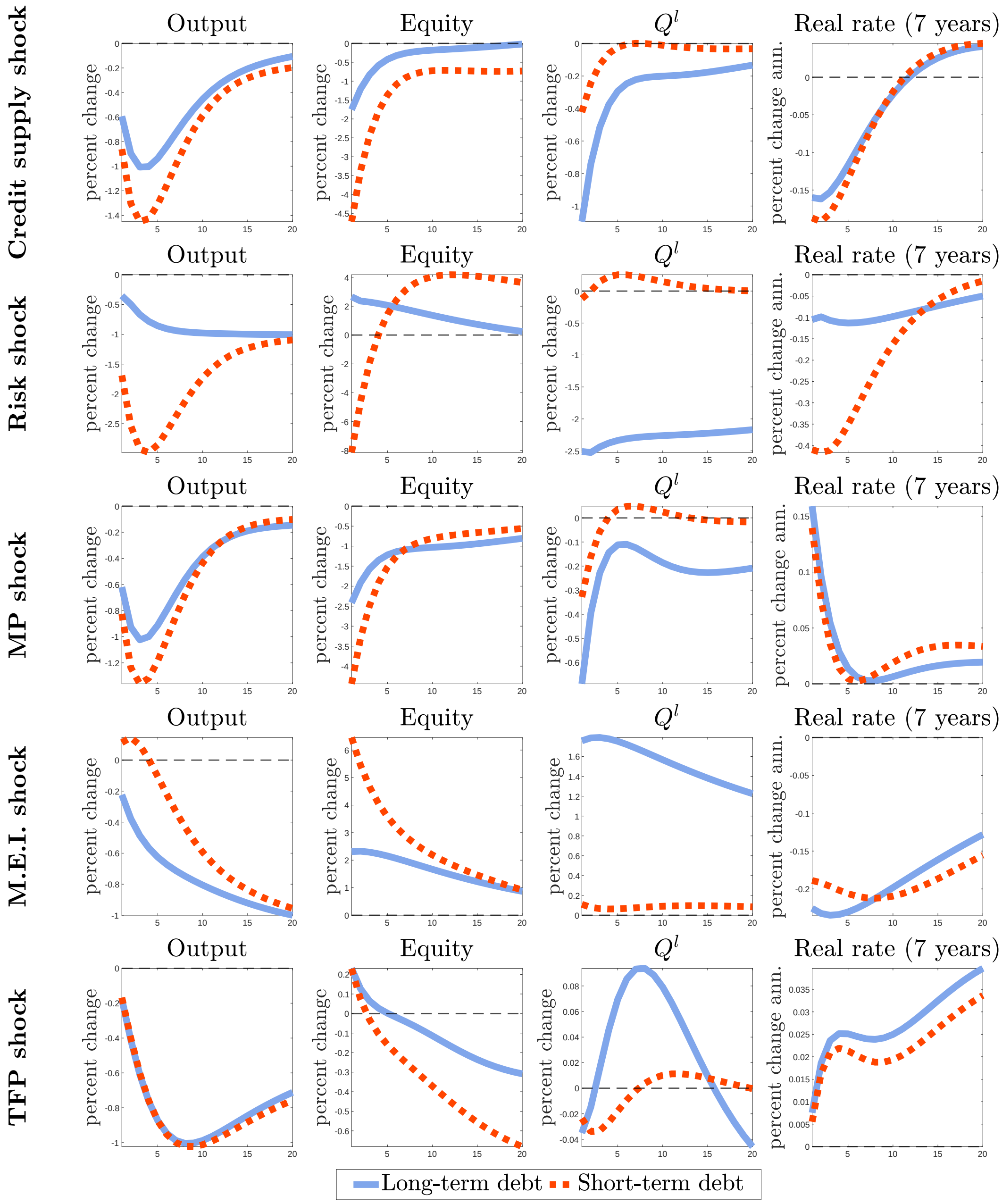
The top three panels of Figure 11 suggest that in a model with realistic equity adjustment costs, long-term debt generates an important attenuation in the transmission of financial shocks which affect investment demand, such as a credit supply shock, a risk shock, or a monetary policy shock. As discussed in Section 3, this result is mainly due to the positive effect of a decline in the price of debt on net worth (balance sheet channel) and to the higher elasticity of leverage (debt dilution channel). Compared to a model with short-term debt, these channels result in a higher path for equity and in a decline in output that is between 30 to 70 percent smaller. The mitigating effect of long-term debt is similar for a credit supply shock and for a contractionary monetary policy shock. The latter result is consistent with the evidence in Jungherr et al. (2024), who show that investment by firms with a larger share of long term debt is less responsive to monetary policy shocks. For the risk shock, the dampening effect of long term debt is particularly strong, and it results in a counterfactual increase in equity which was key for the estimation for the baseline model, as we discussed in the previous section.[33]

The fourth row shows impulse responses to a shock to the marginal efficiency of investment. Unlike the other shocks, long-term debt amplifies the near-term response of output to a marginal efficiency of investment shock. This result is due to the behavior of the price of debt, which increases due to the decline in the real rate, implying a lower path of equity, and hence lower investment and output.

[33] In Figure D.5 we repeat the exercise from Figure 11 while assuming that $\psi = \infty$. When $\psi = \infty$, the gap between the response to financial shocks of the economy with long-term debt and the one with short-term debt becomes even larger, suggesting that the balance sheet channel and the debt dilution channel strengthen as ψ increases.

Finally, the last row shows the impulse response to a TFP shock. In this case debt maturity does not affect the behavior of output in a meaningful way, because equity and the price of debt move very little.

Figure 11: Macroeconomic shocks: The role of long-term debt



Note: Figure compares the impulse responses in our estimated baseline model (blue solid line) to a model with identical parameters except that all debt is short-term (red dashed line). The real rate in the figure is computed as $\mathbb{E}_t \frac{\sum_{i=0}^{27} R_{t+i}/\pi_{t+i+1}}{7}$. All shocks are rescaled to obtain a one percent drop in output in our baseline model with long-term debt.

5 Conclusion

This paper developed and estimated a medium-scale New-Keynesian model with long-term corporate debt and costly equity issuance. Our main findings can be summarized as follows: We showed that the interplay between long-term debt and equity adjustment costs fundamentally alters the transmission of macroeconomic shocks. When equity injections are costly, the debt dilution incentive and the resulting balance-sheet channel both work to partially offset declines in net worth following adverse shocks. This mechanism dampens the contraction of investment and output in response to financial shocks.

Using firm-level balance sheet data, we obtained an estimate of the equity adjustment cost parameter. Incorporating realistic financing frictions into the model led us to a re-assessment of the drivers of business cycle fluctuations. In particular, our framework identified a credit supply shock as the primary source of variability in output and investment, as this is the only shock that generates the observed co-movement between macroeconomic aggregates and financial variables. While here we capture the shock to credit supply in a reduced form way, it would be interesting to extend the analysis and incorporate a model of the financial sector to study the role of financial intermediation more explicitly.

Debt is a key source of financing and the majority of U.S. corporate debt is long-term. Because debt maturity affects both the sensitivity and propagation of shocks, our results suggest that policy makers should consider the underlying corporate financing structure.

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A Model without equity adjustment costs

In this section, we show that in a model with defaultable long-term debt and free equity issuance the equilibrium is not well-defined. By setting $\psi = 0$, we can rewrite the value function of the entrepreneur as

$$v_t\left(\eta_{t-1}, \xi_t\right) = \max_{\eta_t, \omega_t} \left\{ \mu_t\left(\eta_t, \eta_{t-1}, \xi_t\right)\left(1-\omega_t\right) + \mu_t\left(\eta_t, \eta_{t-1}, \xi_t\right) \frac{\omega_t\left(1-\bar{\psi}\right)}{Q_t^k - Q_t^l \eta_t} \mathbb{E}_t \Lambda_{t,t+1} \max\left\{0, v_t\left(\eta_t, \xi_{t+1}\right)\right\} \right\}$$
$$= \mu_t\left(\eta_t, \eta_{t-1}, \xi_t\right) + \omega_t g_t\left(\eta_t\right)$$

where

$$\mu_t\left(\eta_t, \eta_{t-1}, \xi_t\right) = \frac{n_t}{k_{t-1}} = \left[\xi_t R_t^k q_{t-1}^k - \frac{\left(c_l + \left(1-\lambda_l\right) Q_t^l\right)}{\pi_t} \eta_{t-1}\right] \tag{50}$$

$$g_t\left(\eta_t\right) = \mu_t\left(\eta_t, \eta_{t-1}, \xi_t\right)\left(-1 + \frac{\left(1-\bar{\psi}\right) \mathbb{E}_t \Lambda_{t,t+1} \max\left\{0, v_t\left(\eta_t, \xi_{t+1}\right)\right\}}{Q_t^k - Q_t^l \eta_t}\right) \tag{51}$$

where $g_t\left(\eta_t\right)$ represents the net return on an extra unity of equity inside the firm.

The first order conditions for equity and leverage, ω_t^* and η_t^*, are

$$g_t\left(\eta_t^*\right) = 0 \tag{52}$$

$$\frac{d\mu_t}{d\eta_t^*} + \omega_t^* \frac{dg_t\left(\eta_t^*\right)}{d\eta_t^*} = 0 \tag{53}$$

where, we have that

$$\frac{d\mu_t}{d\eta_t^*} = -\left(1-\lambda_l\right) \frac{dQ_t^l}{d\eta_t} \frac{\eta_{t-1}}{\pi_t} > 0 \tag{54}$$

since $\frac{dQ_t^l}{d\eta_t} < 0$ However, these first order conditions necessarily select a saddle point. In fact, equations (53) and (54) imply that $\frac{dg_t(\eta_t^*)}{d\eta_t^*} < 0$. If this is the case, then we can find another an ε such that $\hat{\eta}_t = \eta_t^* - \varepsilon$ and $g_t\left(\hat{\eta}_t\right) > 0$, and set ω_t to infinity to yield an infinite value for the entrepreneur. Basically, the entrepreneur could reduce leverage to increase the excess return on equity, and then freely adjust equity to infinity to obtain an infinite return. Hence, without some increasing marginal costs of adjusting equity, the entrepreneur's problem does not have an interior optimum and consequently the macroeconomic model does not have an equilibrium. Quadratic costs of equity adjustment alter the first order conditions by making the return on equity depend on ω_t, that is $g_t\left(\eta_t, \omega_t\right)$, preventing the entrepreneur from pursuing the alternative strategy described above.

On the other hand, if debt is short term, that is if $\lambda = 1$, an interior optimum would be possible even with $\psi = 0$. In fact, using $\frac{d\mu_t}{d\eta_t^*} = 0$, the first order conditions would become

$$g_t\left(\eta_t^{*,ST}\right) = 0 \tag{55}$$

$$\frac{dg_t\left(\eta_t^{*,ST}\right)}{d\eta_t^{*,ST}} = 0 \tag{56}$$

In this case, equation (56) implies that the optimal leverage is chosen in order to maximize the expected return on equity, preventing any profitable deviation from the interior optimum.

This analysis suggests that in a model with defaultable long-term debt some form of equity adjustment cost is a necessary condition for the existence of the equilibrium. We can think of the model with $\psi = 0$, that we study in section 3 for illustrative purposes, as a limit case with arbitrarily small equity adjustment costs.

An additional result for the model with $\psi = 0$ is that the derivative of the entrepreneurial leverage policy function is zero, that is $\eta_\eta = 0$. This can be seen by rewriting equation (52) as

$$\frac{1}{\left(1-\bar{\psi}\right)} = \frac{\mathbb{E}_t\Lambda_{t,t+1}\max\left\{0, v_t\left(\eta_t, \xi_{t+1}\right)\right\}}{Q_t^k - Q_t^l\eta_t} \tag{57}$$

which is equivalent to equation (41) in the paper. In this case, equation (57) pins down η_t as a function of only the aggregate price of capital and the household stochastic discount factor. Leverage η_t will not depend on past leverage η_{t-1} or ω_t and hence $\eta_\eta = 0$. The equation (53) will instead determine ω_t.

B Algorithm to compute derivative of leverage

In our baseline model, the entrepreneur's individual choice of equity (ω_t^i) and leverage (η_t^i) is characterized by the following first order conditions

$$1 = \Psi' \left(\omega_t^i\right) \gamma_t^i \tag{58}$$

$$\begin{gathered}\left(Q_t^{l,i} + \frac{dQ_t^{l,i}}{d\eta_t^i}\eta_t^i\right)\gamma_t^i - \frac{(1-\lambda_l)\eta_{t-1}^i}{\pi_t}\frac{dQ_t^{l,i}}{d\eta_t^i}\frac{Q_t^k - Q_t^{l,i}\eta_t^i}{\Psi\left(\omega_t^i\right)\mu_t^i}\varphi_t^i \\ = \\ \mathbb{E}_t\Lambda_{t,t+1}\frac{c_l+(1-\lambda_l)Q_{t+1}^{l,i}}{\pi_{t+1}}\varphi_{t+1}^i\left(1 - F_t\left(\bar{\xi}_{t+1}^i, \sigma_t\right)\right)\end{gathered} \tag{59}$$

where

$$Q_t^{l,i} = \mathbb{E}_t\Lambda_{t+1}\kappa_t\left\{\left[1 - F_t\left(\bar{\xi}_{t+1}^i, \sigma_t\right)\right]\frac{c_l + (1-\lambda_l)\,Q_{t+1}^{l,i}}{\pi_{t+1}} + \gamma_l\frac{Q_t^k R_{t+1}^k}{\eta_t^i}\int_0^{\bar{\xi}_{t+1}^i}\xi_{t+1}dF_t\left(\xi_{t+1}, \sigma_t\right)\right\} \tag{60}$$

$$\begin{aligned}\frac{\partial Q_t^{l,i}}{\partial\eta_t^i} = &-\mathbb{E}_t\Lambda_{t+1}\kappa_t\left\{\frac{\partial\bar{\xi}_{t+1}^i}{\partial\eta_t^i}f_t\left(\bar{\xi}_{t+1}^i\right)\left[Q_t^{l,i}R_{t+1}^{l,i} - \gamma_l\frac{Q_t^k R_{t+1}^k}{\eta_t^i}\bar{\xi}_{t+1}^i\right] + \gamma_l\frac{Q_t^k R_{t+1}^k}{\left(\eta_t^i\right)^2}\int_0^{\bar{\xi}_{t+1}^i}\xi_{t+1}dF_t\left(\xi_{t+1}\right)\right\} \\ &+ \mathbb{E}_t\Lambda_{t+1}\kappa_t[1 - F_t\left(\bar{\xi}_{t+1}^i\right)]\left(1-\lambda_l\right)\frac{\partial Q_{t+1}^{l,i}}{\partial\eta_{t+1}^i}\eta_{\eta,t+1}\left(\eta_t^i\right)\end{aligned} \tag{61}$$

$$\gamma_t^i = \frac{1}{Q_t^k - Q_t^{l,i}\eta_t^i}\mathbb{E}_t\Lambda_{t,t+1}\varphi_{t+1}^i\int_{\bar{\xi}_{t+1}^i}\left(\xi_{t+1} - \bar{\xi}_{t+1}^i\right)R_{t+1}^k Q_t^k dF_t\left(\xi_{t+1}, \sigma_t\right) \tag{62}$$

$$\varphi_t^i = \left\{\left(1 - \omega_t^i\right) + \Psi\left(\omega_t^i\right)\gamma_t^i\right\} \tag{63}$$

$$\bar{\xi}_{t+1}^i = \frac{\left(c_l + (1-\lambda_l)\,Q_{t+1}^{l,i}\right)}{\pi_{t+1}}\frac{\eta_t^i}{R_{t+1}^k Q_t^k} \tag{64}$$

$$\mu_t^i = \tilde{\xi}_t R_t^k Q_{t-1}^k - \frac{c_l + (1-\lambda_l)\,Q_t^{l,i}}{\pi_t}\eta_{t-1}^i, \tag{65}$$

These equations determine a policy function for *individual* leverage which we denote as $\eta^i(\eta_{t-1}^i, S_t, \sigma^\epsilon)$, where η_{t-1}^i is the *individual* leverage at time $t-1$, S_t is the aggregate state of the economy, which also include the *aggregate* level of leverage at time $t-1$, and σ^ϵ is a vector collecting the standard deviation of all exogenous shocks. Notice that this policy function is different from the policy function for *aggregate* leverage.

Our algorithm to approximate $\frac{\partial \eta^i(\eta^i_{t-1}, S_t, \sigma^\epsilon)}{\partial \eta^i_{t-1}}$ works in two steps. In the first step, we compute $\eta^i(\eta^i_{t-1}, S^{ss}, 0)$ and its derivative $\frac{\partial \eta^i(\eta^i_{t-1}, S^{ss}, 0)}{\partial \eta^i_{t-1}}$ in the model steady state. This is done with the following iterative procedure:

- Given an initial guess for S^{ss} and associated prices and quantities in steady state, and for $\eta_{\eta,ss}$, we use global methods to solve for $\eta^i(\eta^i_{t-1}, S^{ss}, 0)$ over a grid on entrepreneurial leverage η^i_{t-1}.

- Given the policy function $\eta^i(\eta^i_{t-1}, S_{ss}, 0)$, we can compute $\eta_\eta = \frac{\partial \eta^i(\eta^i_{t-1}, S_{ss}, 0)}{\partial \eta^i}$ and update $\eta_{\eta,ss}$ and S_{ss}.

- These steps are repeated until convergence.

In the second step of our algorithm we approximate the derivative function denoted by $g(\eta^i_{t-1}, S_t, \sigma^\epsilon) = \frac{\partial \eta^i(\eta^i_{t-1}, S_t, \sigma^\epsilon)}{\partial \eta^i_{t-1}}$ linearly around the steady state. To do so we write the policy function for leverage in an economy without aggregate risk, as an explicit function of previous leverage, aggregate prices at time t, expected prices at $t+1$, and all other aggregate quantities that the entrepreneur takes as given in the system (58)–(65). That is, letting $\Gamma_t = \{\eta^i_{t-1}, \tilde{\xi}_t, \sigma_t, \kappa_t, \mathbb{E}_t R^k_{t+1}, R^k_t, Q^k_t, Q^k_{t-1}, \mathbb{E}_t \Lambda_{t+1}, \mathbb{E}_t \pi_{t+1}, \pi_t\}$ we denote by $\tilde{\eta}(\Gamma_t)$ be the policy function that solves the optimality conditions above without aggregate uncertainty and for a level of previous leverage, prices at time t, expected prices at $t+1$, and all other aggregate quantities given by Γ_t. We let $\tilde{g}(\Gamma_t) = \frac{\partial \tilde{\eta}}{\partial \eta_{t-1}}$ and our approximation be given by

$$\hat{g}(\Gamma_t) = \tilde{g}(\Gamma^{ss}) + \nabla \tilde{g} d\Gamma_t$$

where the gradient of $\tilde{g}$ is computed numerically by solving the policy functions for $\tilde{\eta}$ globally for every small deviation of each argument in Γ_t from steady state.

C Estimation of equity adjustment costs

This appendix provides more detail on the data treatment, estimation, and robustness of the results presented in Section 4.2.

We perform the following cleaning steps on our quarterly Compustat sample. We remove firms in the finance, government, and utility sectors. We remove non-US firms. Observations with negative total assets are dropped. We transform cumulative variables `sstkyl`, `dvy`, and `prstkcy` into quarterly variables.

Our dependent variable is $R^x_{i,t+1} = \frac{V_{i,t+1}}{X_{i,t}}$, where $V_{i,t+1}$ is the $t+1$ cum-dividend market value of equity. It is constructed using Compustat variables `cshoq`×`prccq` + `netpayouts`, where `netpayouts` = `dvyq` - (`sstkyq` - `prstkcyq`). We remove negative observations of $V_{i,t+1}$. The denominator $X_{i,t}$ is the cum-dividend book value of equity, defined as `ceqq` + `netpayouts`. Again, we remove negative observations.

Our key independent variable is $\omega_{it} = 1 - \frac{\texttt{netpayouts}}{X_{i,t}}$, i.e., one minus the fraction of net payouts to shareholders over the cum-dividend book value of equity. We remove observations of ω_{it} that include negative values of $X_{i,t}$.

Finally, we trim the independent variable at the 5th and 95th percentile. The dependent variable is truncated at zero from below, we therefore only trim it at the 95th percentile.[34]

The regression we run is

$$\frac{R^x_{i,t+1}}{1 + r_{t+1}} = \psi \omega_{i,t} + \delta_i + \delta_t + \nu_{it}, \tag{66}$$

where r_{t+1} is the quarterly realized real rate and δ_i and δ_t are firm- and time fixed effects.

Our results are shown in Table A1. The first column shows the regression coefficient of the model estimated in (66). Columns (2) and (3) show the effect of removing the firm or time fixed effects.

Table A1: Estimating the equity adjustment cost parameter

	(1)	(2)	(3)	(4)	(5)	(6)
ω	10.86	15.45	11.77	10.78	15.31	11.69
	(0.13)	(0.15)	(0.13)	(0.13)	(0.15)	(0.13)
Firm fixed effects	yes	no	yes	yes	no	yes
Time fixed effects	yes	yes	no	yes	yes	no
Additional cleaning	no	no	no	yes	yes	yes
N	512,318	512,848	512,318	494,201	494,651	494,201

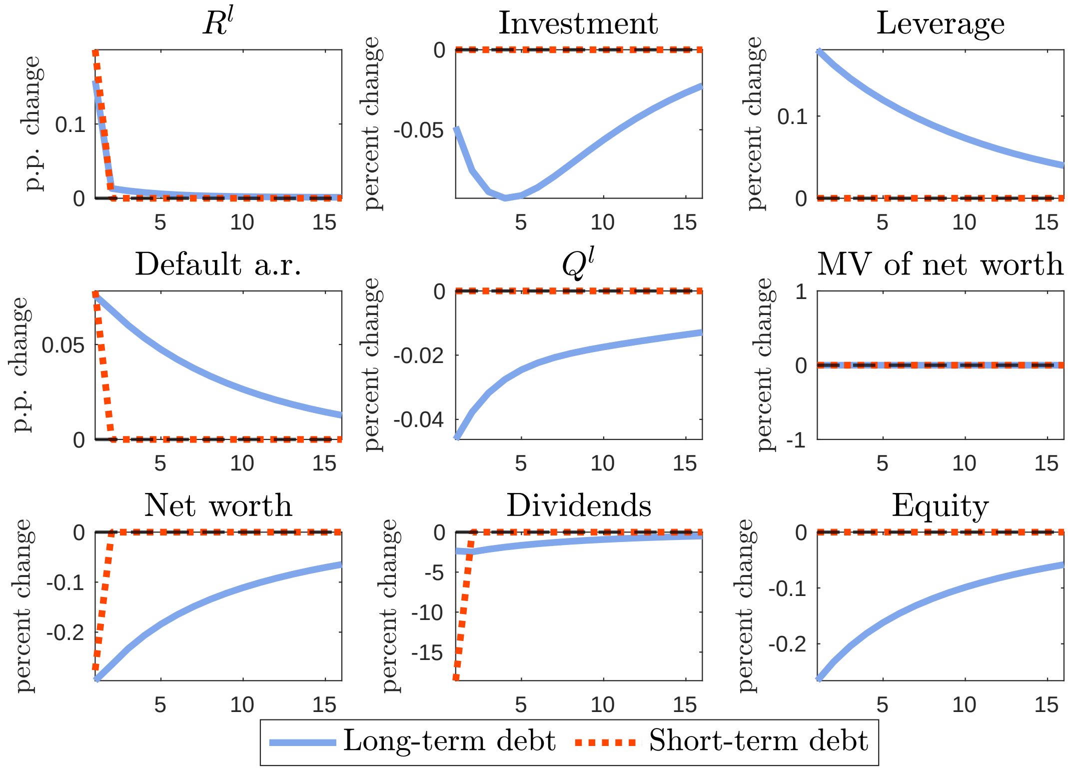
[34] Winsorizing instead of trimming the data leads to similar results.

Robustness As a robustness exercise, we perform additional cleaning steps. We only keep firms with at least five quarters of information in the sample. Further, we remove firm-quarters that violate the accounting identity by more than 10 per cent of the book value of assets (as in Covas and Den Haan (2012)). Lastly, we drop firms involved in major mergers (Compustat footnote `saleq_fn1` code "AB"). As indicated in columns (4)–(6) of Table A1, these additional cleaning steps hardly affect our results.

D Additional model experiments

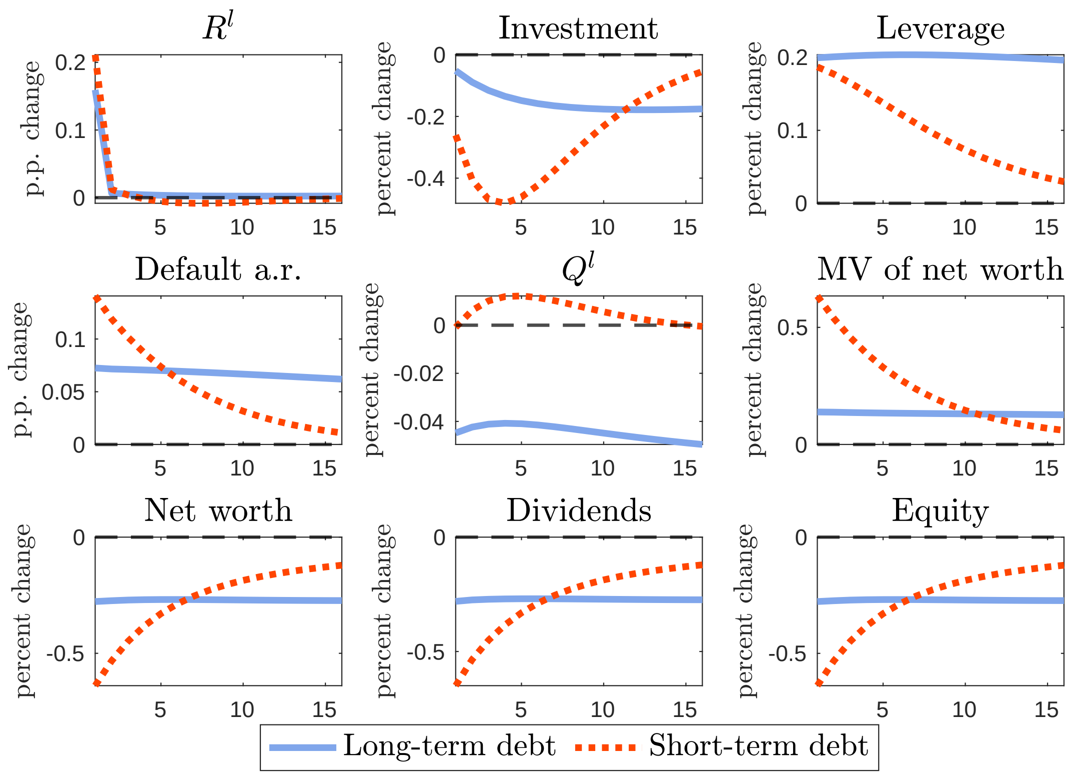
This appendix provides additional impulse response functions and variance decompositions for the model with long-term debt and equity adjustment costs. Figures D.1 and D.2 show the impulse responses to a debt shock in the baseline model without equity adjustment costs and with infinite equity adjustment costs, respectively. Figure D.3 shows the impulse responses to a net worth shock in the baseline model. Figure D.4 compares the impulse responses in the baseline model with those in a model with $\psi = \infty$. Table A2 shows the variance decomposition for that case. Finally, Figure D.5 compares the impulse responses when debt is short- or long-term in the case of $\psi = \infty$.

Figure D.1: Debt shock in model with sticky prices: no equity adjustment costs



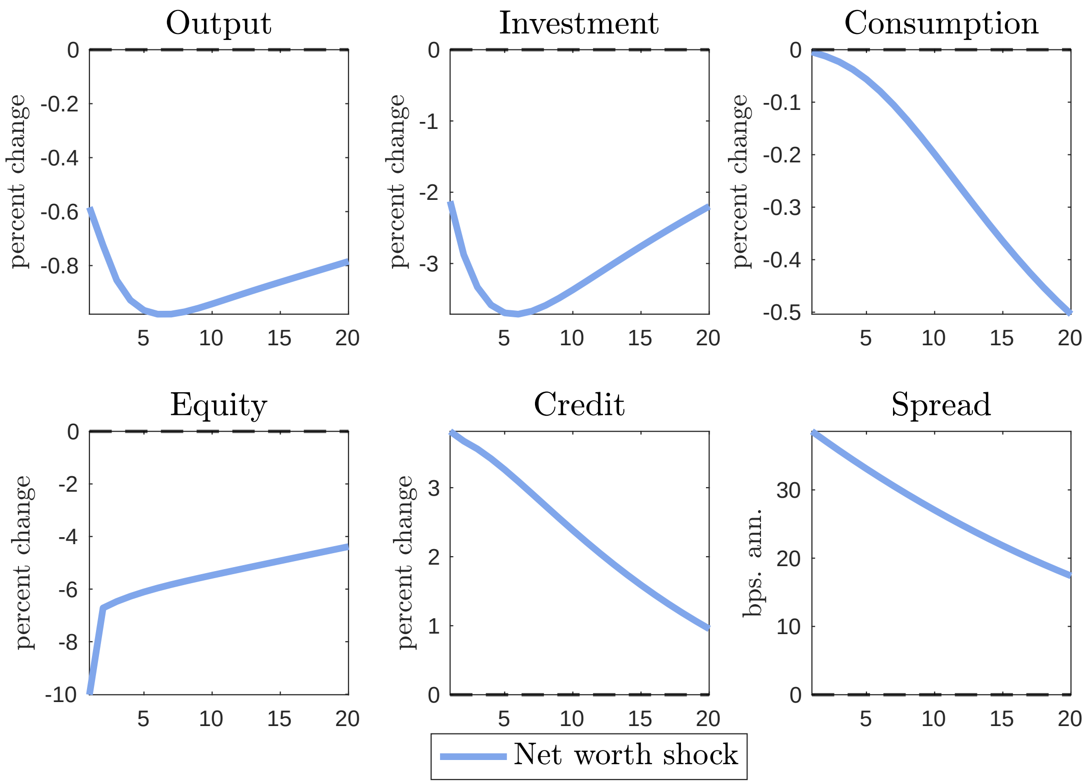
Note: Figure shows the impulse response to a one-time shock to the value of outstanding debt in a version of the baseline model with no real default costs, i.e. $\nu = 0$.

Figure D.2: Debt shock in model with sticky prices: infinite equity adjustment costs



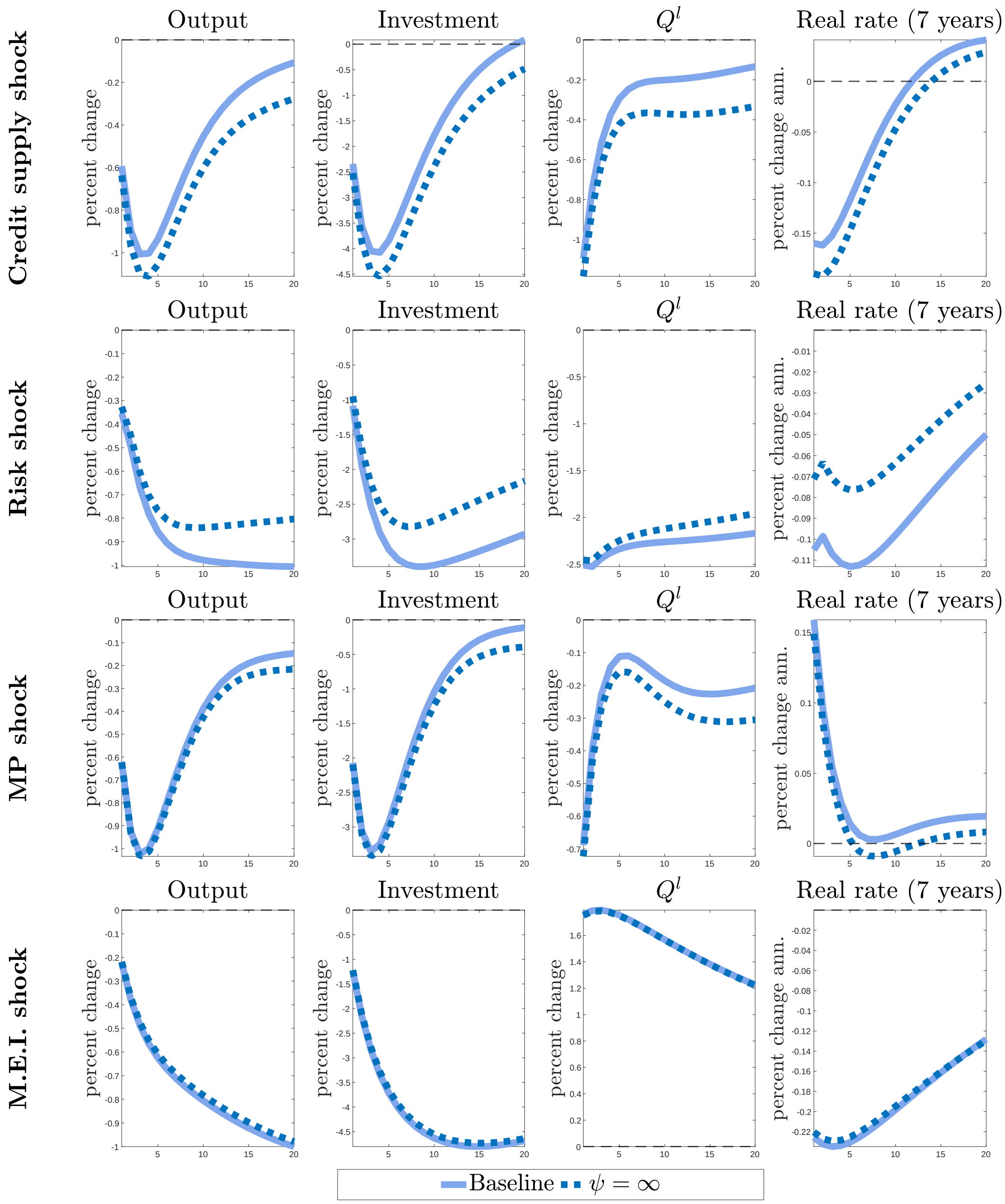
Note: Figure shows the impulse response to a one-time shock to the value of outstanding debt in a version of the baseline model with infinite equity adjustment costs no real default costs, i.e. $\nu = 0$.

Figure D.3: Net worth shock in baseline model



Note: Figure shows the impulse response to shock to the entrepreneurial net worth in our baseline model

Figure D.4: Impulse responses in baseline model vs model with $\psi = \infty$

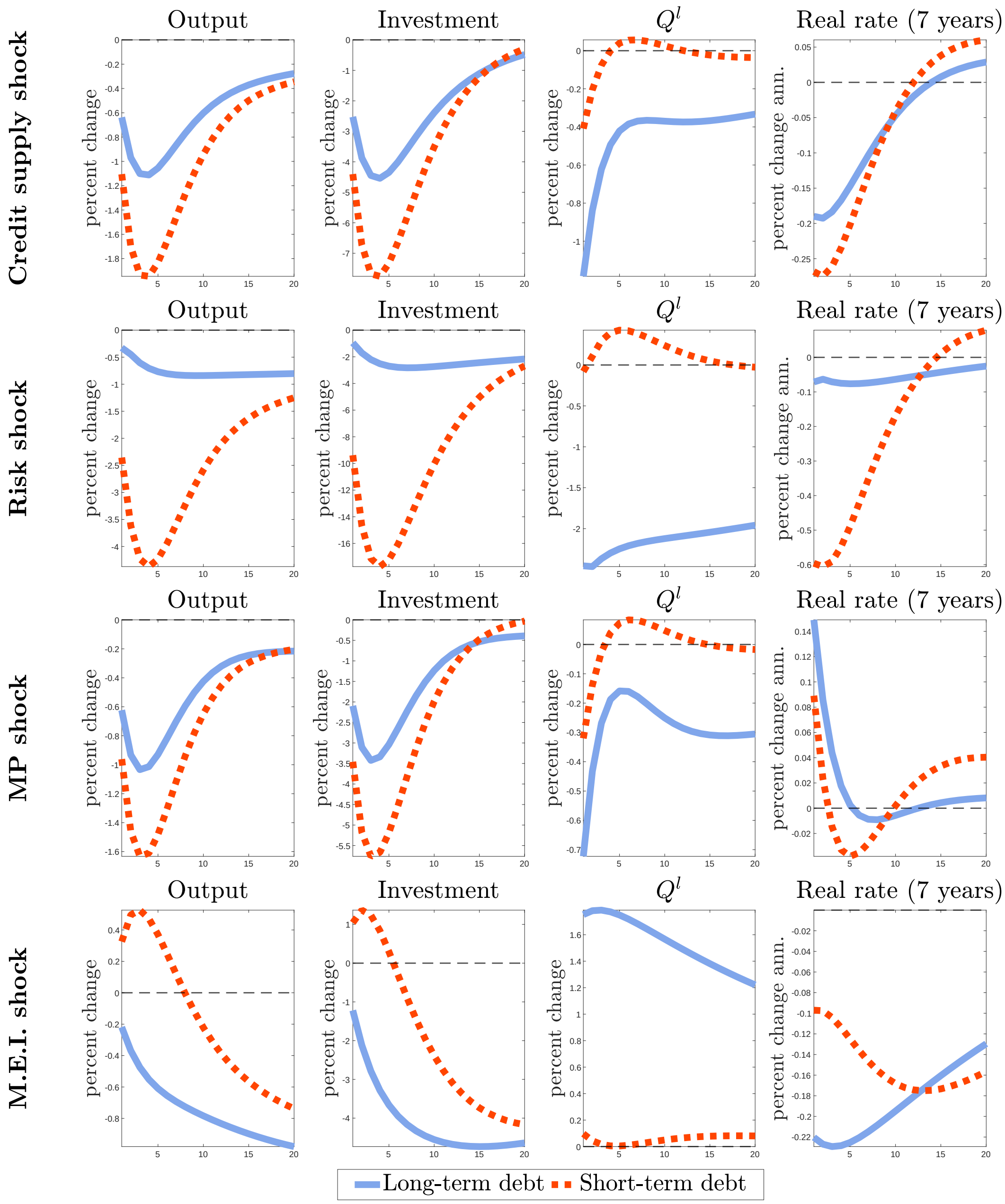


Note: Figure compares the impulse responses in our baseline model (blue solid line) to a model with the same parameters but with $\psi = \infty$ (dark blue dashed line).

Table A2: Variance Decomposition: Model with $\psi = \infty$

Model with long-term debt and $\psi = \infty$								
	Credit Supply	Risk	M.E.I.	NW	Markup	Gov	M.P.	TFP
GDP	46	4	6	5	18	8	4	4
Investment	54	4	15	6	13	0	3	2
Consumption	7	3	25	1	2	1	0	4
Spread	6	62	21	9	0	0	0	1
Equity	15	6	16	58	1	0	2	0
Credit	21	12	35	27	1	0	1	0
Inflation	38	7	10	1	21	1	5	5
FFR	48	10	28	3	1	1	2	1
Hours	26	10	37	2	14	2	2	1
Wages	2	1	4	0	10	0	0	3

Figure D.5: Long-term debt vs short-term debt in baseline model with $\psi = \infty$



Note: Figure compares the impulse responses in our baseline model (blue solid line) and in a model with short-term debt (red dashed line), in both cases assuming that $\psi = \infty$.